**1.** Describe how to construct a surjective immersion $\mathbb{R}^2 \to S^2$.

**2.** Show that the punctured torus $S^1 \times S^1 - \{point\}$ has an immersion into $\mathbb{R}^2$. Include some explanation of why your construction is $C^\infty$. [Hint for the construction: enlarge the puncture.]

**3.** Show that a $C^\infty$ homeomorphism $M \to N$ whose inverse is $C^1$ is a diffeomorphism.

**4.** Show that any product of (finitely many) spheres can be embedded into Euclidean space of one higher dimension.

**5.** Show that if $M$ is a smooth compact $n$-manifold, $n > 0$, then any smooth map $f\colon M \to \mathbb{R}$ has at least two critical points.

**6.** Show that $\mathbb{R}\mathrm{P}^2$ and the Klein bottle can both be smoothly embedded in $\mathbb{R}^4$. [Hint: for $\mathbb{R}\mathrm{P}^2$, decompose it as the union of a Möbius band $M$ and a disk $D$ with their boundary circles identified. Embed $M$ and $D$ separately in $\mathbb{R}^3$ with a common boundary circle, and so that their union $\mathbb{R}\mathrm{P}^2$ is immersed, then push the interiors of $M$ and $D$ out into opposite sides of $\mathbb{R}^3$ in $\mathbb{R}^4$.]

**7.** (a) Let the torus $T$ be embedded in $\mathbb{R}^3$ in the standard way, say as the surface of revolution obtained by rotating the circle $(x-2)^2 + z^2 = 1$ in the $xz$-plane about the $z$-axis. Determine all the critical points and critical values of the projection $f\colon T \to \mathbb{R}^2$, $f(x,y,z) = (x,y)$.

(b) What are the critical points and critical values if we project the same embedded $T$ onto the $xz$-plane instead of the $xy$-plane?

(c) Construct an embedding of $T$ in $\mathbb{R}^3$ such that the critical points and critical values of the projection $T \to \mathbb{R}^2$ each consist of $n$ disjoint circles, for arbitrary $n \geq 2$.