Hints for 8.2, 8.3 and 8.4

1. (8.3.57) Evaluate $A = \int x^3\sqrt{1-x^2}dx$ using
a. integration by parts
We use IBP for $u = x^2$, $dv = x\sqrt{1-x^2}$ $du = 2x$, $v = -\frac{1}{3}(1-x^2)^{3/2}$ then
$A = -\frac{x^2}{3}(1-x^2)^{3/2} + \frac{1}{3}\int \frac{1}{3}(1-x^2)^{3/2}2xdx = -\frac{x^2}{3}(1-x^2)^{3/2} - \frac{2}{15}(1-x^2)^{5/2} + C$
b. a u-substitution
Let $u = 1-x^2$ then $du = -2xdx$ so $x^3dx = \frac{1}{2}x^2 2xdx = -(1-u)du$. Thus,

$$A = -\frac{1}{2}\int(1-u)\sqrt{u}du = -\frac{1}{2}\int(u^{1/2} - u^{3/2})du$$
$$= -\frac{1}{3}u^{3/2} + \frac{1}{5}u^{5/2} + C = -\frac{1}{3}(1-x^2)^{3/2} + \frac{1}{5}(1-x^2)^{5/2} + C$$

c. a trig substitution
$x = \sin\theta$ for $-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$ and proceed as in class

4. (8.3.35)Calculate $A = \int_0^{\ln 4} \frac{e^t dt}{\sqrt{e^{2t}+9}}$
First let $u = e^t$ then $du = e^t dt$ and the boundary points are $e^0 = 1$ and $e^{\ln 4} = 4$ then
$A = \int_1^4 \frac{du}{\sqrt{u^2+9}}$
Now we use the trig substitution $u = 3\tan\theta$ for $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ then
$\sqrt{u^2+9} = 3|\sec\theta| = 3\sec\theta$ as $\sec\theta$ is nonnegative on the domain.
Furthermore, $du = 3\sec^2\theta d\theta$. Thus
$\int \frac{du}{\sqrt{u^2+9}} = \int \frac{3\sec^2\theta d\theta}{3\sec\theta} = \int \sec\theta d\theta = \ln|\sec\theta + \tan\theta| + C$
Now in order to evaluate the definite integral, we have to write down that last result in term of u.
$\tan\theta = \frac{u}{3}$ and since $\sec\theta$ is nonnegative,

$$\sec\theta = \sqrt{1+\tan^2\theta} = \sqrt{1+\frac{u^2}{9}}$$

Then $A = (\ln\sqrt{1+\frac{u^2}{9}} + \frac{u}{3})\,|_1^4 = \ln 9 - \ln(\sqrt{10}+1)$

6. First doing a long division yields

$$\frac{x^3}{x^2-2x+1} = x + 2 + \frac{x-2}{x^2-2x+1}$$

Then since $x^2 - 2x + 1 = (x-1)^2$, $\frac{x-2}{x^2-2x+1} = \frac{A}{x-1} + \frac{B}{(x-1)^2}$.
Multiplying both sides by $(x-1)^2$ and then letting $x = 1$ give $B = -1$.
Substituting into the above equation gives $A = 1$