Hints for 9.1, 9.2, and 7.2

3. In order to solve a 1st order **linear** DE, we set $v = e^{\int P dx}$. Suppose H_1 and H_2 are two antiderivative of $P(x)$, show that $v_1 = e^{H_1}$ and $v_2 = e^{H_2}$ would give the same result for the solution of the DE.

$H_1 = H_2 + C$ and thus $v_1 = Av_2$. Plug in the formula of the solution we see the A is cancelled out.

4a. $\tan(x)y' + y = \frac{\sin x}{1-\sin^2 x}$, $-\pi/2 < x < \pi/2$

Apply the method for first order linear DE.
The final answer is $y = \frac{1}{\sin x}(\ln(\sec x) + C)$ for $x \neq 0$, $y = 0$ for $x = 0$. Pay special attention that at some point you have to be careful if $x = 0$ so it is better to treat that case separately.

4b. $\frac{dy}{dx} = e^{x-y} + e^x + e^{-y} + 1$, $y(0) = 2$

Notice that $e^{x-y} + e^x + e^{-y} + 1 = (e^x + 1)(e^{-y} + 1)$ and use the method for separable DE