Hints for 10.3 and 10.4

2. Use the integral test to compare the series with the integral $\int_2^\infty \frac{dx}{x\ln x\sqrt{(\ln x)^2+1}}$.
Then use the substitution $u = \ln x$ to transform the integral into $\int_{\ln x}^\infty \frac{du}{u\sqrt{u^2+1}}$
Finally, use the limit comparision test to compare $\frac{1}{u\sqrt{u^2+1}} \sim \frac{1}{u^2}$.
The last integral converges so the original one also converges.
3. Either direct comparision test or limit comparision test with the geometric series $b_n = \frac{6^n}{7^n}$

4. For $n > 1$, $\frac{1}{n!} \leq \frac{1}{n(n-1)}$ (here $n(n-1)$ are the last two terms in $n!$). Then we can use the limit comparison test between $a_n = \frac{1}{n(n-1)}$ and $b_n = \frac{1}{n^2}$ to conclude that all series here are convergent.

5. Using the limit comparision test between $a_n = \frac{1}{\sqrt{n}\ln n}$ and $b_n = \frac{1}{n}$ with $\lim_{n\to\infty} \frac{a_n}{b_n} = \infty$. Then apply the conclusion of the LCT for the case $L = \infty$ and $\sum_{n=1}^\infty b_n$ convergent.
Again this example shows the hierarchy:

$$\ln << \text{polynomial functions} << \text{exponential functions}$$