Warmup4 Week of Sep 17th

1. When considering $\int \sin^m x \cos^n x \; dx$, does n being odd make things easier or harder?

1. Harder since for the even case, you just need to remember one formula, the double angle one.
2. It doesn't really matter - the answer is $\frac{\sin^{m+1} x}{m+1} \frac{\cos^{n+1} x}{n+1}$ anyway
3. Yes, because then $u = \sin x$ combined with $\cos^2 x = 1 - \sin^2 x$ will convert the integral to the integral of a polynomial
4. Yes, because integration by parts will now easily give the answer

2. What would happen if the integrand in example 4 of section 8.2 changed from $\sqrt{1 + \cos 4x}$ to $\sqrt{2 + \cos 4x}$?

1. After a substitution $u = 4x$ one could use the same method.
2. Similar difficulty. You'd start by simplifying the square root.
3. Much harder since the half angle formula would not let you quickly get rid of the square root.
4. Similar in difficulty. All examples in the text are examples of general methods.

3. The substitution $x = \sin u$ would be useful for

1. $\int \sqrt{x^2 - 1} \; dx$ since $\sin^2 u = 1 - \cos^2 u$.
2. $\int \sqrt{x^2 + 1} \; dx$ using $1 + \tan^2 u = \sec^2 u$.
3. $\int \sqrt{-x^2 + 1} \; dx$ using $\cos^2 u = 1 - \sin^2 u$.

4. Suppose we used the substitution $x = \sec u$ with $x < 0$ and $\frac{\pi}{2} < u < \pi$ even though our textbook prefers not to. Then

1. $\sqrt{x^2 - 1}$ would simplify to $\tan u$ as usual.
2. $\sqrt{x^2 - 1}$ would simplify to the square root of a negative number.
3. $\sqrt{x^2 - 1}$ would simplify to $-\tan u$ and we'd be stuck.
4. $\sqrt{x^2 - 1}$ would simplify to $-\tan u$ and there'd be no further problem.

5. Is there anything wrong with using the (Heaviside) "cover up" method to find B in $\frac{x^3}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$?

1. No, just cover up the $(x + 1)$ on the left and plug in $x = -1$ to get $B = 4$.
2. Yes, because we haven't divided out x^3 by $x^2 - 1$ first.
3. Yes you need to cover up the $(x + 1)$ on the left and plug in $x = 1$ instead of $x = -1$.