#### MATH 1340 — Mathematics & Politics

Lecture 10 — July 6, 2015

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## Arrow's Theorem

<u>Arrow's Impossibility Theorem (1950):</u> If a social choice function with at least three candidates satisfies Pareto and independence, then it is (functionally equivalent to) a dictatorship.

<u>Corollary:</u> It is impossible for a social choice function with at least three candidates to satisfy Pareto, independence and anonymity.

• For the proof, see section 5.4 of R&U or your notes from class.

# A different way?

- One response to Arrow's theorem is to settle: Plurality (or Copeland, or...?) is good enough, and independence is too much to hope for.
- Another response is to start over: Why do we need to use preference ballots?
- If we replace our preference ballots with something different, can we do better?

## Approval ballots (a brief tour)

- An **approval ballot** is a ballot in which a voter express their "yes" (Y) or "no" (N) approval of each candidate.
- An **approval profile** is the list of all approval ballots from all of the voters in a given electorate.

Α	Y	Y	Ν	Ν	Y	Ν	
В	Y	Ν	Y	Y	Y	Ν	Approval profile
С	Ν	Ν	Y	Y	Y	Ν	
▼							-

Approval ballot of voter 3

• Tabulated approval profiles can also be defined.

#### Approval social choice functions

- An approval social choice function is a function having domain all approval profiles of a fixed electorate, and codomain *all* subsets of the slate of candidates.
  - Unlike with preference ballots, we allow the possibility of outputting the empty set; i.e., choosing no winners. This may be reasonable when all of the voters disapprove of all of the candidates.
- The plurality (approval) method is the approval social choice function that selects as the winner(s) whichever candidate(s) get the most "yes" votes.
  - This method always outputs some winner, even if every vote is N (then, all of the candidates tie).
  - It is used to elect the Secretary General of the United Nations, as well as the presidents of several scientific societies including the Mathematical Association of America and the American Mathematical Society.

## Approval social choice functions (cont'd)

- We say that a candidate A is socially preferred to B (in a given profile) if the number of approval votes for A is greater than the number of approval votes for B.
- In the following way, approval ballots avoid the Condorcet paradox:

<u>Proposition:</u> If in a given approval profile, A is socially preferred to B, and B is socially preferred to C, then A is socially preferred to C.

Why? "Greater than" is itself transitive.

### Criteria for approval methods

Here are some desirable criteria for approval social choice functions:

- An approval social choice function is **anonymous** if the outcome is unchanged whenever two voters exchange approval ballots.
- An approval social choice function is **neutral** if whenever A is a winner in one profile, and all voters who approved of A but not some other candidate B change to approve of B but not A, and and all voters who approved of B but not A change to approve of A but not B (everything else remains unchanged), then B becomes a winner.
- An approval social choice function is **monotone** if whenever a winner gets more "yes" votes, they remain a winner.
- An approval social choice function is **nearly decisive** if whenever it names two winners, they must have exactly the same number of "yes" votes.

# Criteria for approval methods (cont'd)

- An approval social choice function is **independent** if whenever we are given two profiles in which every voter has the same opinion of candidates A and B, and A wins and B loses in the first profile, then B must not win in the second profile.
- This criterion is substantially less "mysterious" in the approval ballot setting.
- One can check (it's not too hard):

<u>Proposition:</u> The plurality method is anonymous, neutral, monotone, nearly decisive, and independent.

## Criteria for approval methods (cont'd)

• In fact, more is true:

<u>Theorem:</u> The only approval social choice function that is anonymous, neutral, monotone, independent, nearly decisive, and that always selects at least one winner is the plurality method.

• See p. 102 of R&U for the details.

- Recommended reading: Section 5.4 in R&U
- Optional reading: Chapter 6 (specifically 6.3 on Approval Ballots) and the "Notes on Part I" in R&U
- Problem set #4 is due tomorrow, in class.
- Test 1 on Voting and Social Choice Theory is on Wednesday, in class. Please review the end of the slides from Lecture 9 for details. (Note: Approval voting is **not** part of the test material.)