MATH 1340 — Mathematics & Politics

Lecture 15 — July 13, 2015

Gerrymandering

Variation on “The Gerry-mander”,
Boston Gazette, March 26, 1812

From apportionment to redistricting

- The method (Hill's) by which we *apportion* congressional seats to the states has been fixed in law since 1941.

- Despite some problems (e.g., quota violations, claims of bias against large states), apportionment has remained largely uncontroversial in recent decades.

- The method by which those seats are distributed *within* each state (**districting**, or **redistricting**), however, is *extremely* controversial.

From apportionment to redistricting (cont'd)

- Once the number of representatives for a state is determined by apportionment, it is largely up to that state how to elect those representatives, as per:

  "The times, places and manner of holding elections for Senators and Representatives, shall be prescribed in each state by the legislature thereof; but the Congress may at any time by law make or alter such regulations, except as to the places of choosing Senators."

  -Article I, Section 4 of the Constitution of the United States of America

From apportionment to redistricting (cont'd)

- Since 1967, states which are apportioned more than one representative have been required to be divided into **districts** (i.e., physical regions which partition the state), each of which must hold its own election for a representative using the plurality method.

- Prior to this, there were some instances of states electing their representatives as blocks, without regard to geography.

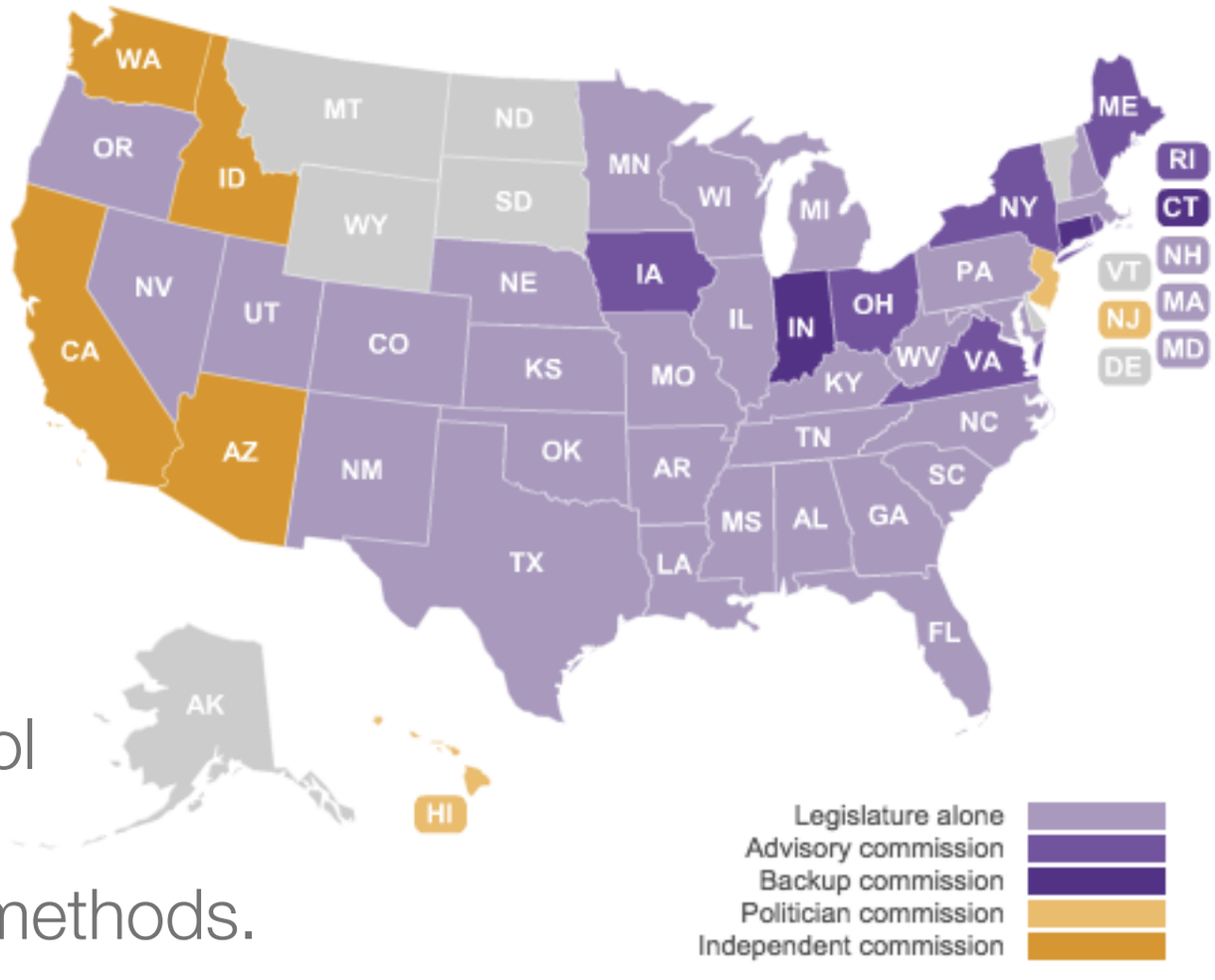
- Aside: Senators, which are now elected by plurality vote every six years in every state, used to be appointed by state legislatures. This ended with the passage of the 17th Amendment in 1913.

Who draws congressional districts?

- Under the terms of the Constitution, state legislatures are entitled to drawing congressional districts, and take the sole role of doing so in most states.

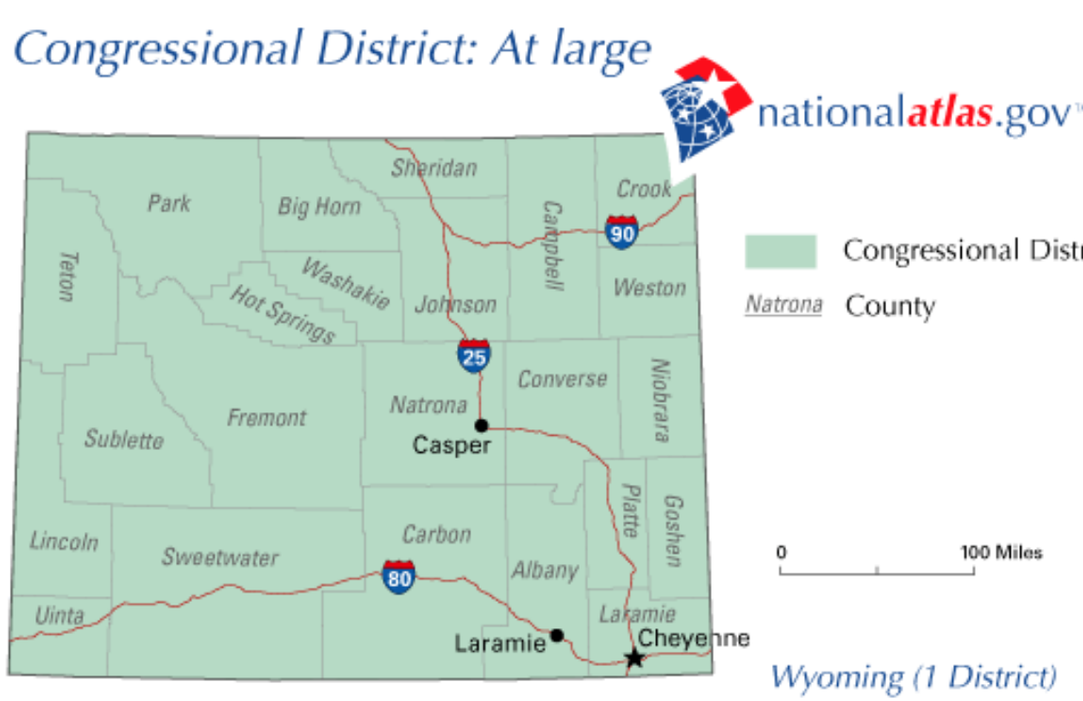
- Some states have opted to have independent commissions draw their districts, while others have advisory commissions, though the final decision is still made by the legislature.

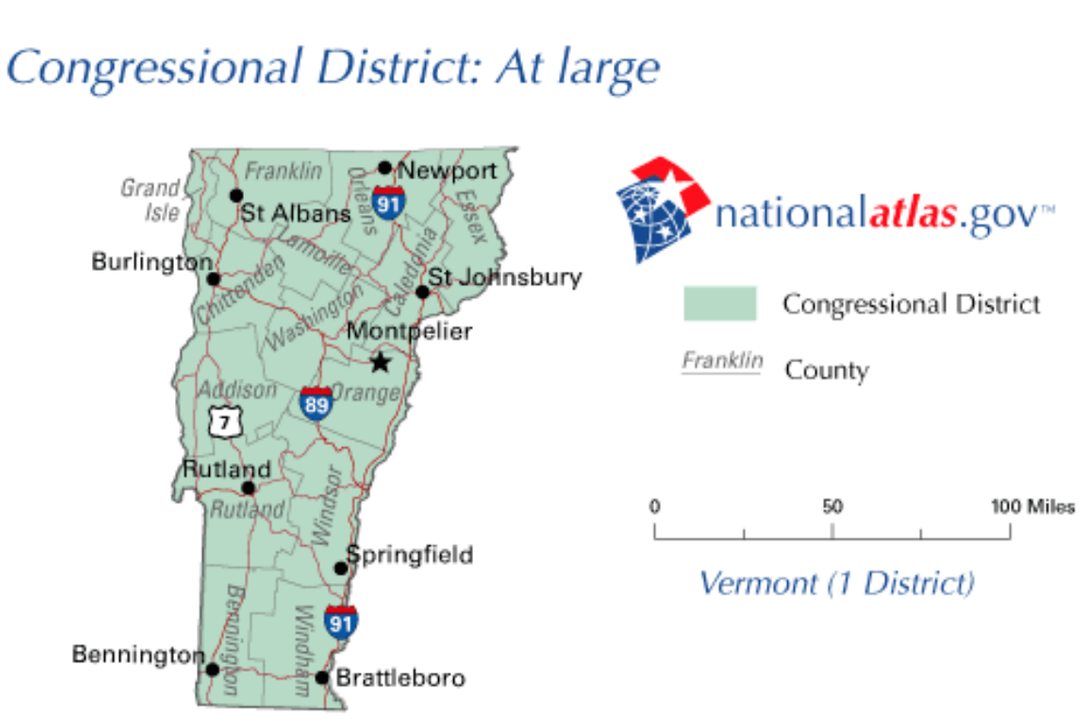
- This map (from Loyola Law School professor Justin Levitt) shows which states currently use which methods.



Rules for drawing congressional districts

- In states with only one representative (Alaska, Delaware, Montana, North Dakota, South Dakota, Vermont, Wyoming), it's easy: the whole state is the one and only district (called an "at-large" district).





Rules for drawing congressional districts (cont'd)

- In the other states, there are many legal restrictions. The easiest to describe are as follows:

- In Wesberry v. Sanders (1963), the Supreme Court ruled that the Equal Protection Clause of the 14th Amendment demands that districts must be (roughly) the same size in population within each state. (A violation of this is called **malapportionment**.)
  - This was frequently violated (with ratios as high as 3 to 1) prior to the Civil Rights Era of the 1960s by southern states which did not redraw their district for decades, causing overrepresentation of rural (predominately white) areas, and underrepresentation of urban (more black) areas.

- Districts must be **contiguous**: A person must be able to walk between any two points within the district while remaining in the district.

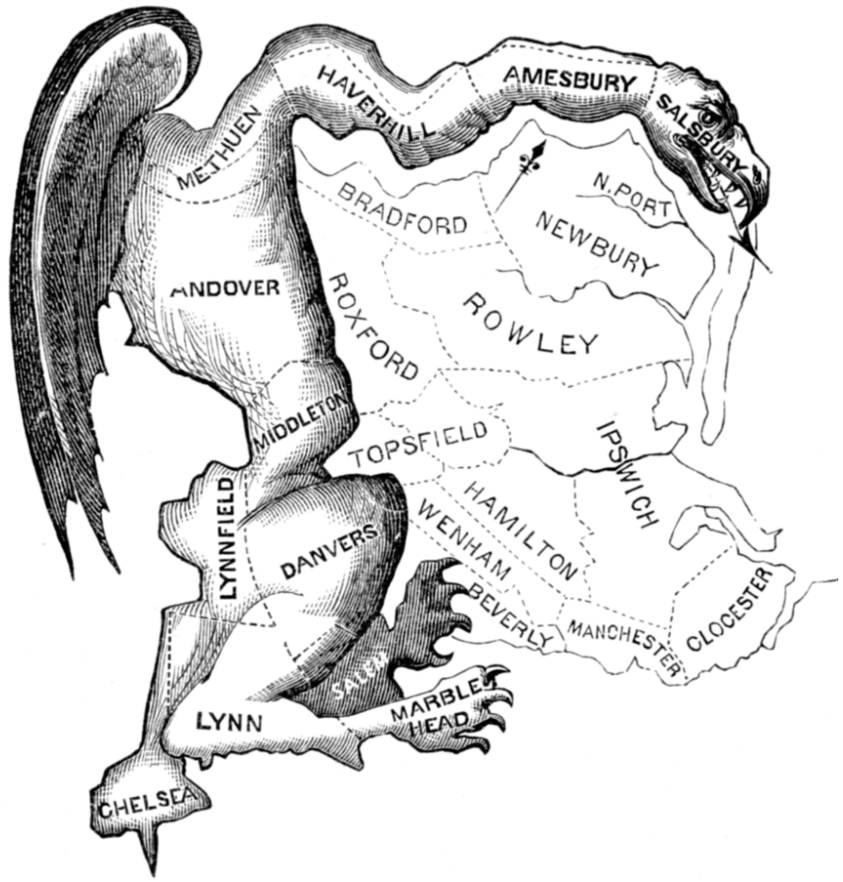
Rules for drawing congressional districts (cont'd)

- Other restrictions on districts are harder to describe, detect and enforce:
- Districts must be **compact**: there is no satisfactory definition of this.
- Districts must respect **communities of interest** such as neighborhoods, minority communities, etc.
- In Shaw v. Reno (1992), the Supreme Court ruled that the 14th Amendment demands that districts must not be drawn with *racial* concerns as the "predominant factor".
- However, *partisan* concerns, while theoretically "justiciable" (Davis v. Bandemer, 1985), have yet to be found sufficient reason for invalidating a district (e.g., it was explicitly allowed in Hunt v. Cromartie, 2000).
- The Voting Rights Act (1965) mandates that certain states with a history of discrimination must "pre-clear" their redistricting plans with the US Dept. of Justice, however the method by which states qualify for this was ruled unconstitutional by the Supreme Court in Shelby County v. Holder (2012). Since this has yet to be amended by congress, this provision is effectively void.

Gerrymandering

- **Gerrymandering** is the act of purposefully drawing district lines to favor one political group/party over others.

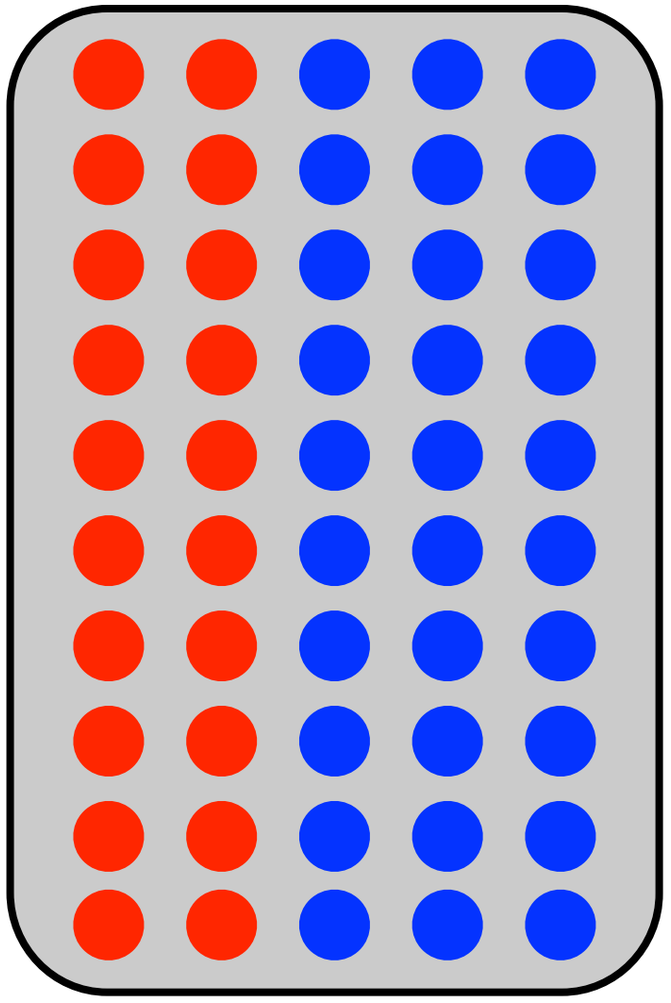
- It is named for Massachusetts Governor Elbridge Gerry, who in 1812 approved a map for state senate districts which contained one oddly shaped district, believed to be drawn to favor his Democratic-Republican Party.



- Its shape was likened to a monster and a salamander by commentators, resulting in the portmanteau “Gerry-mander”.

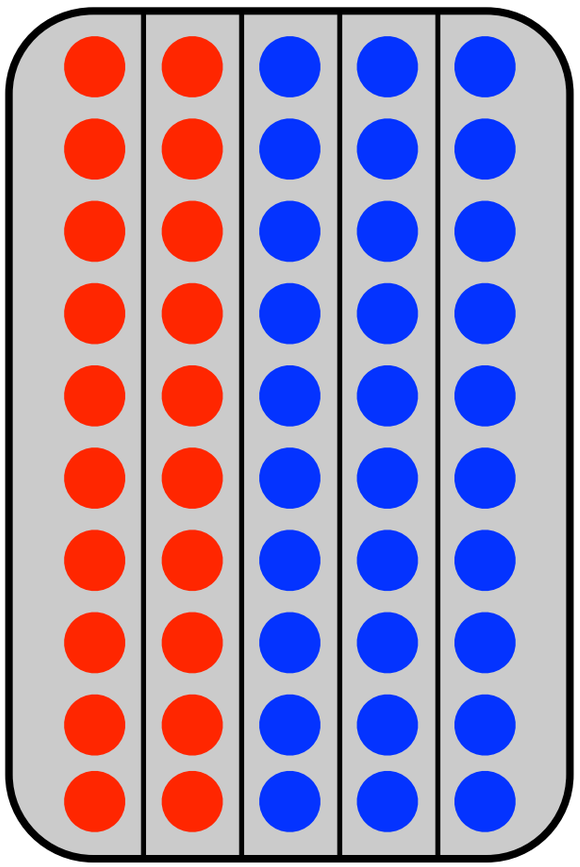
Gerrymandering (cont'd)

- Consider the following "state" (adapted from this Washington Post article), in which there are 50 people, 30 of which are "blue" and 20 of which are "red".

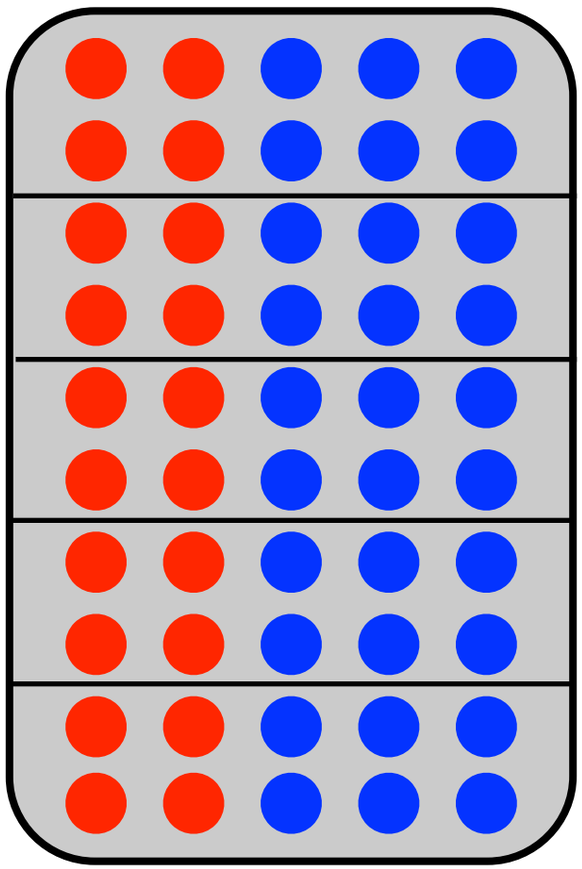


- Can you draw 5 districts, of 10 people each which yields:
  (a) 3 blue, 2 red districts?
  (b) 5 blue districts?
  (c) 2 blue, 3 red districts?

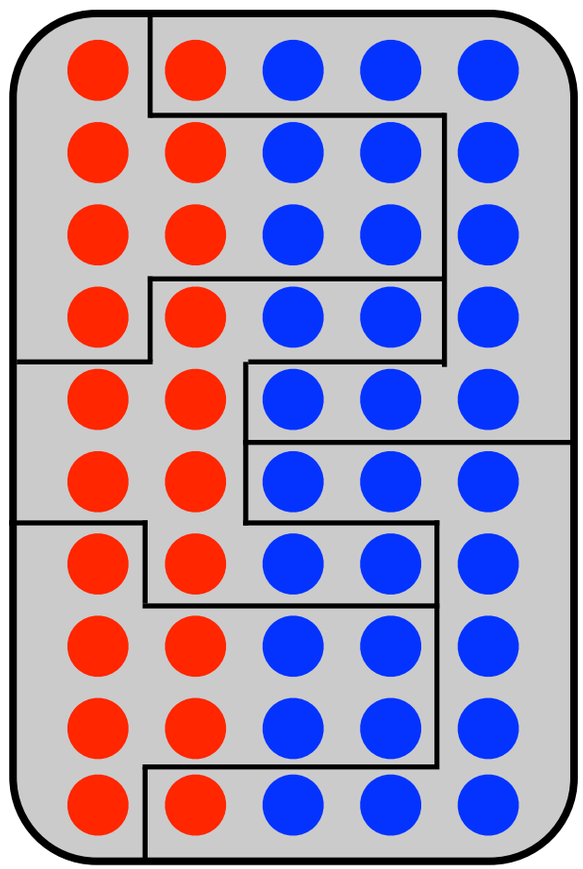
Gerrymandering (cont'd)



3 blue, 2 red



5 blue, 0 red



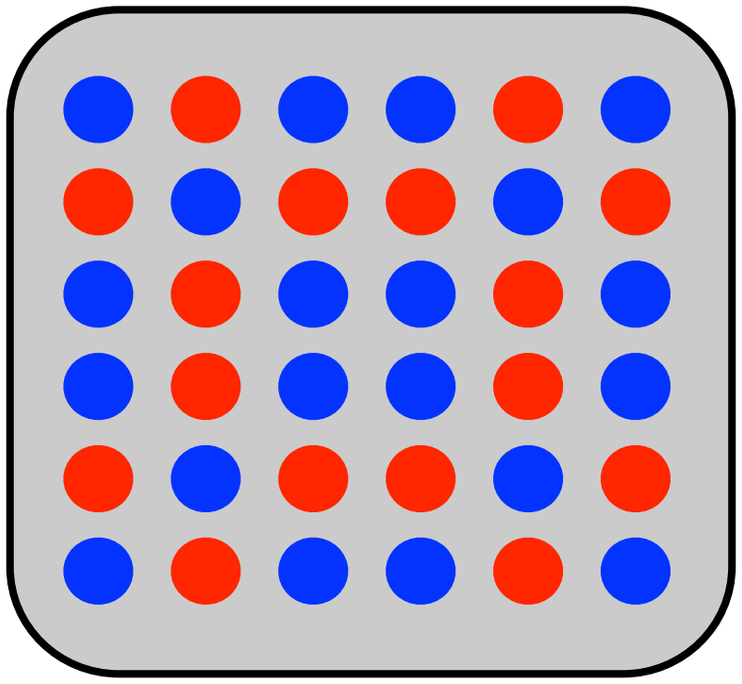
2 blue, 3 red

- The second example demonstrates **cracking**: spreading out your opponents into several districts, diluting their power.

- The last example demonstrates **packing**: placing a large majority of your opponents into a small number of districts which they win easily, but giving you a large number of districts in which you win by a smaller majority.

Gerrymandering (cont'd)

- Consider the following "state" (adapted from this article), in which there are 36 people, 20 of which are "blue" and 16 of which are "red".



- Can you draw 4 districts, of 9 people each which yields :
  (a) 4 blue districts?
  (b) 1 blue, 3 red districts?

Gerrymandering (cont'd)

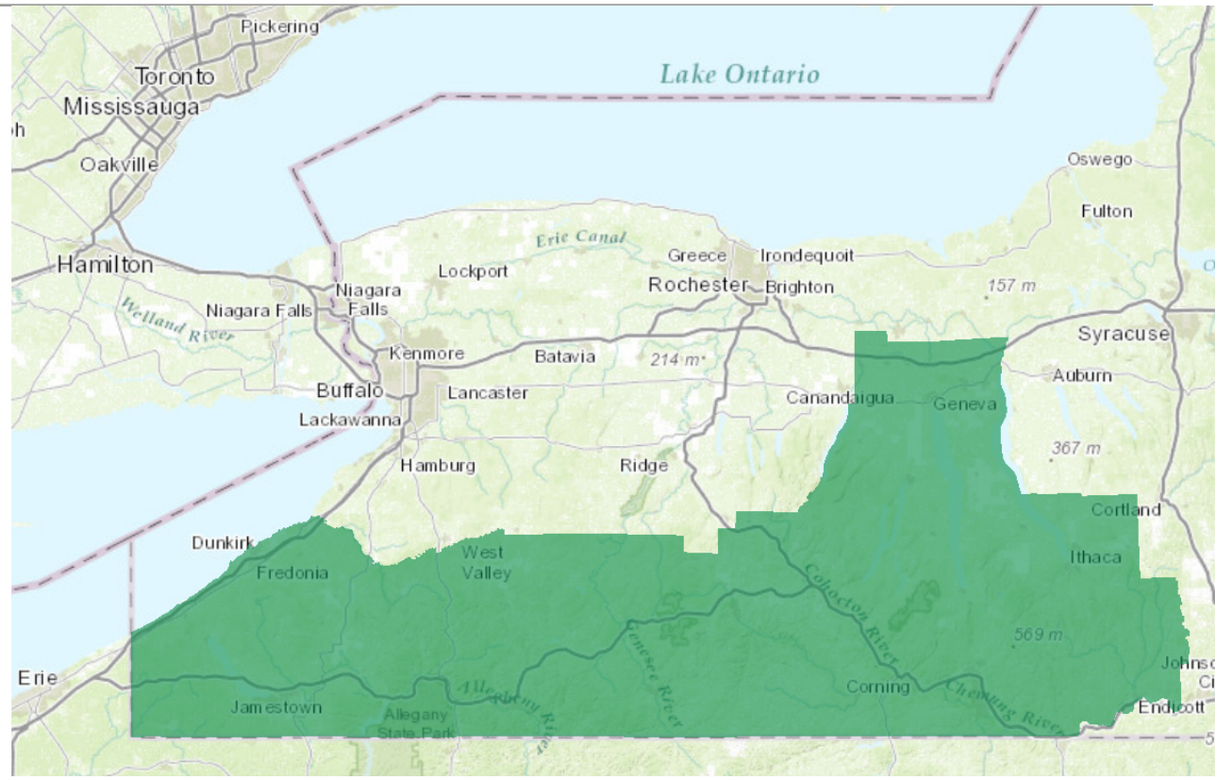
- Partisan legislators have an incentive to gerrymander state districts to favor their party.

- Gerrymandering is often blamed for congressional representation that is not proportionate to the popular vote within states.
  - For example, in 2012, Democrats received 66% of the popular vote for representatives in NY, but they won 21 out of 27 (78%) House seats.
  - In 2012, Republicans received 49% of the popular vote for representatives in PA, but they won 13 (!) out of 18 (72%) House seats.
  - Overall, in 2012, Democrats received 1,711,566 (around 1.7%) *more* votes than Republicans for representatives, but Republicans won 234 House seats, to the Democrats' 201.

- Gerrymandering is also blamed for wild and strange shapes of many congressional districts, as we shall see.

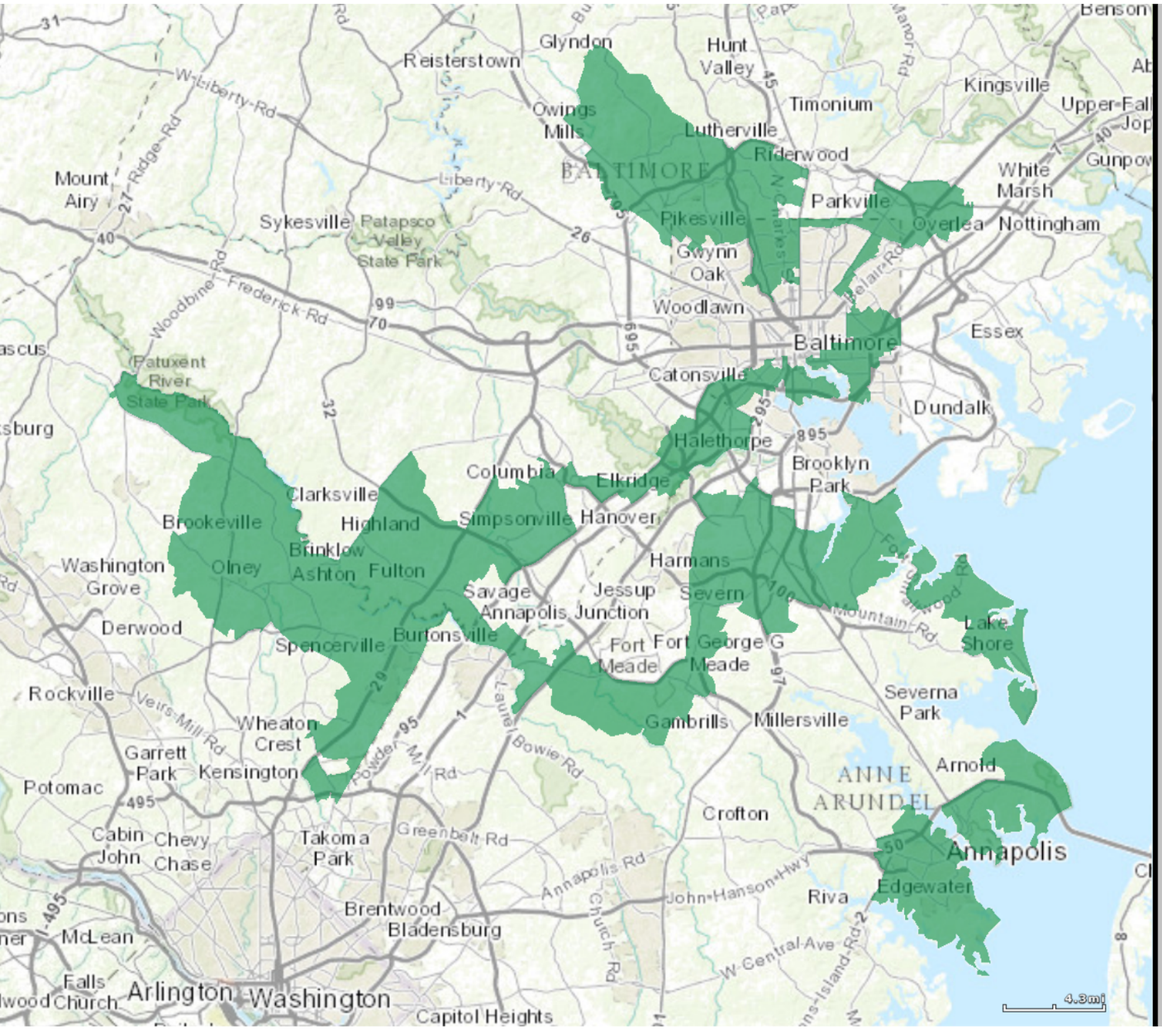
Examples: Ithaca's district

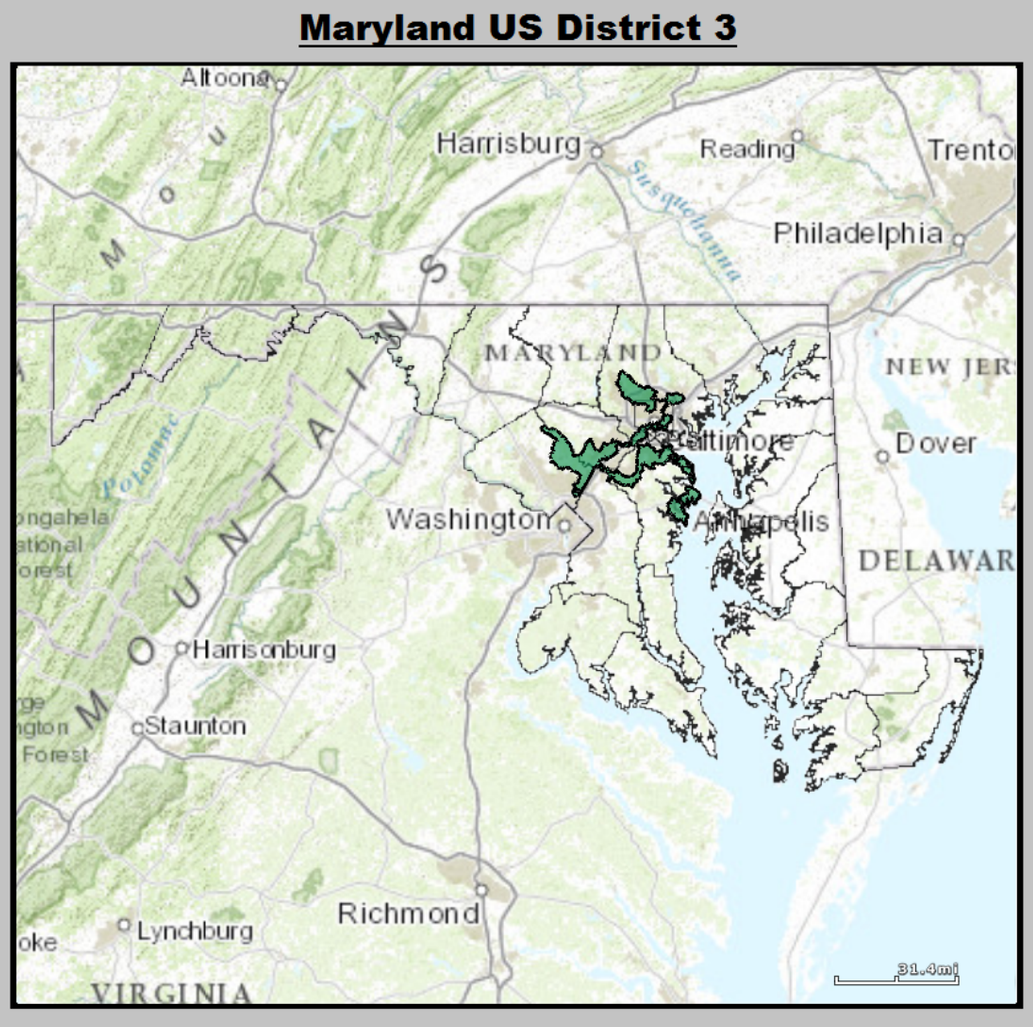
- Ithaca's current district, NY 23:



- Ithaca's old district, NY 22:

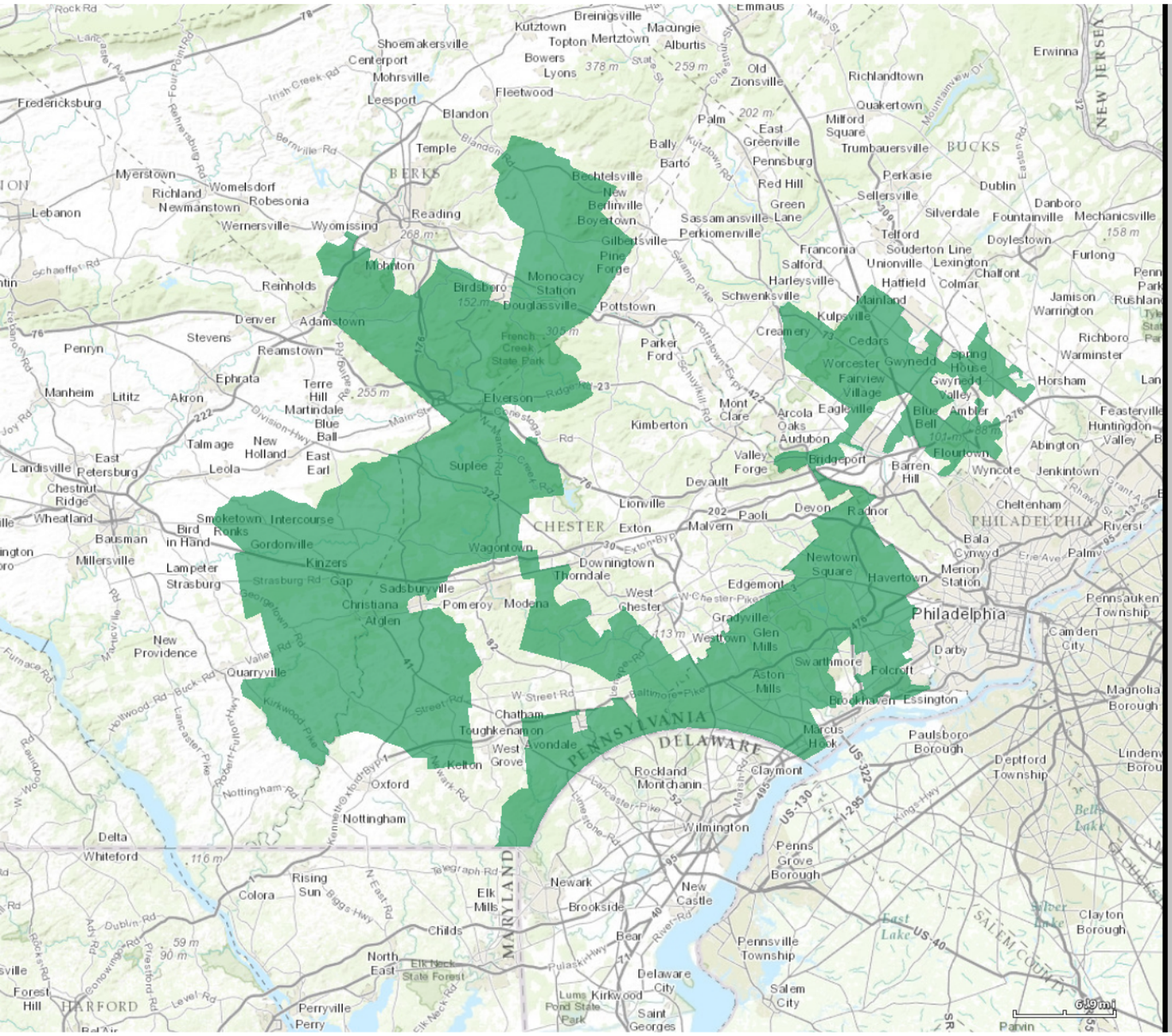
Examples: Maryland's 3rd District





US Congressional districts since 2013
Source: http://nationalatlas.gov, 1 Million Scale project.

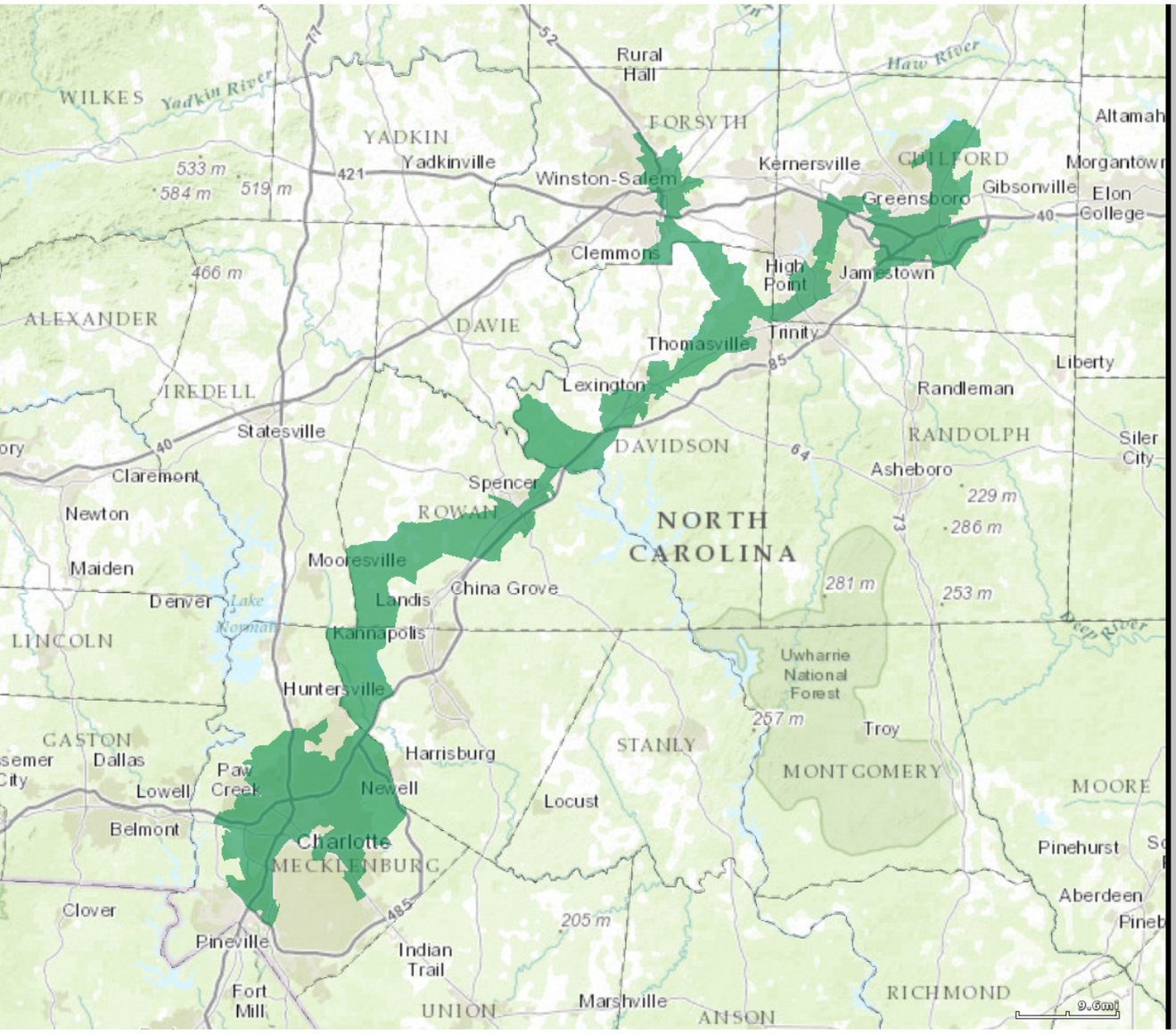
Examples: Pennsylvania's 7th District



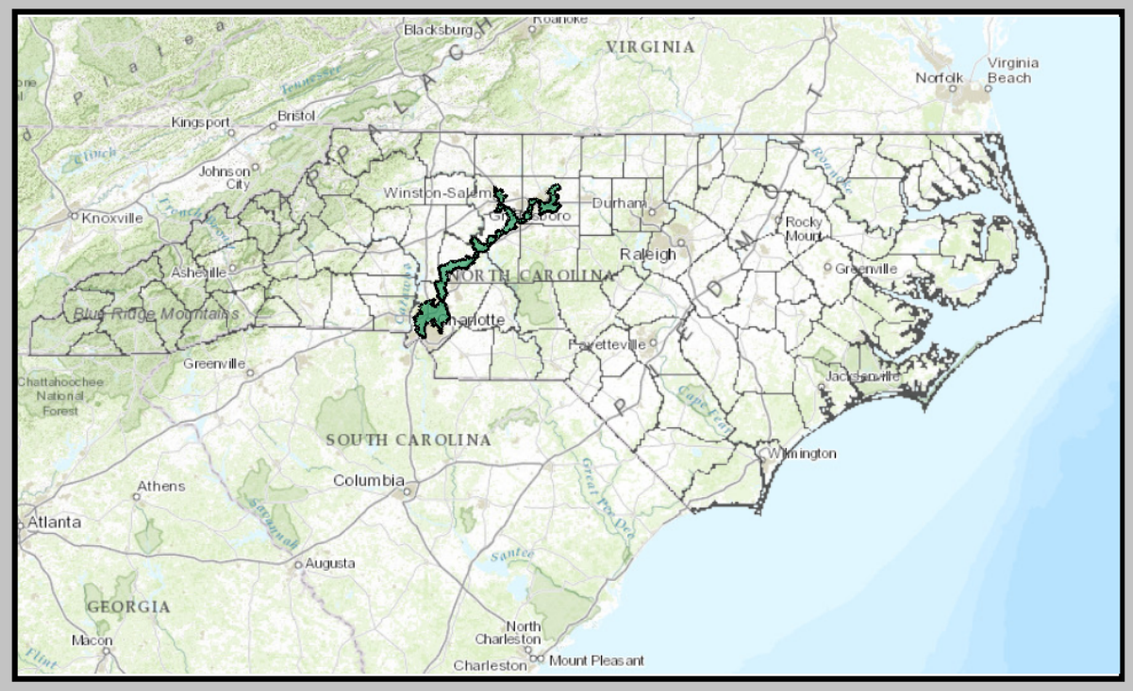
Pennsylvania US District 7

US Congressional districts since 2013
Source: http://nationalatlas.gov, 1 Million Scale project.

Examples: North Carolina's 12th District

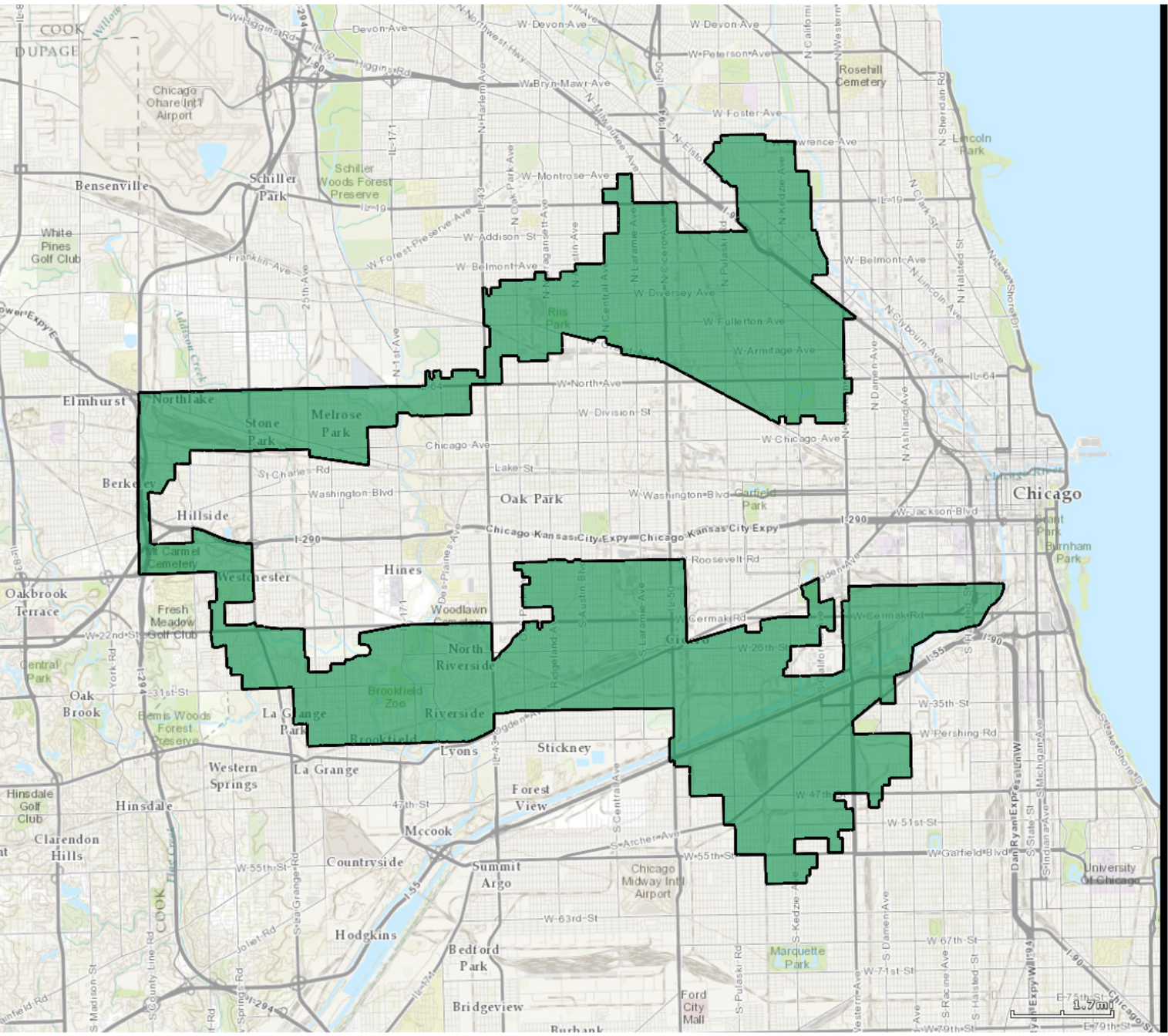


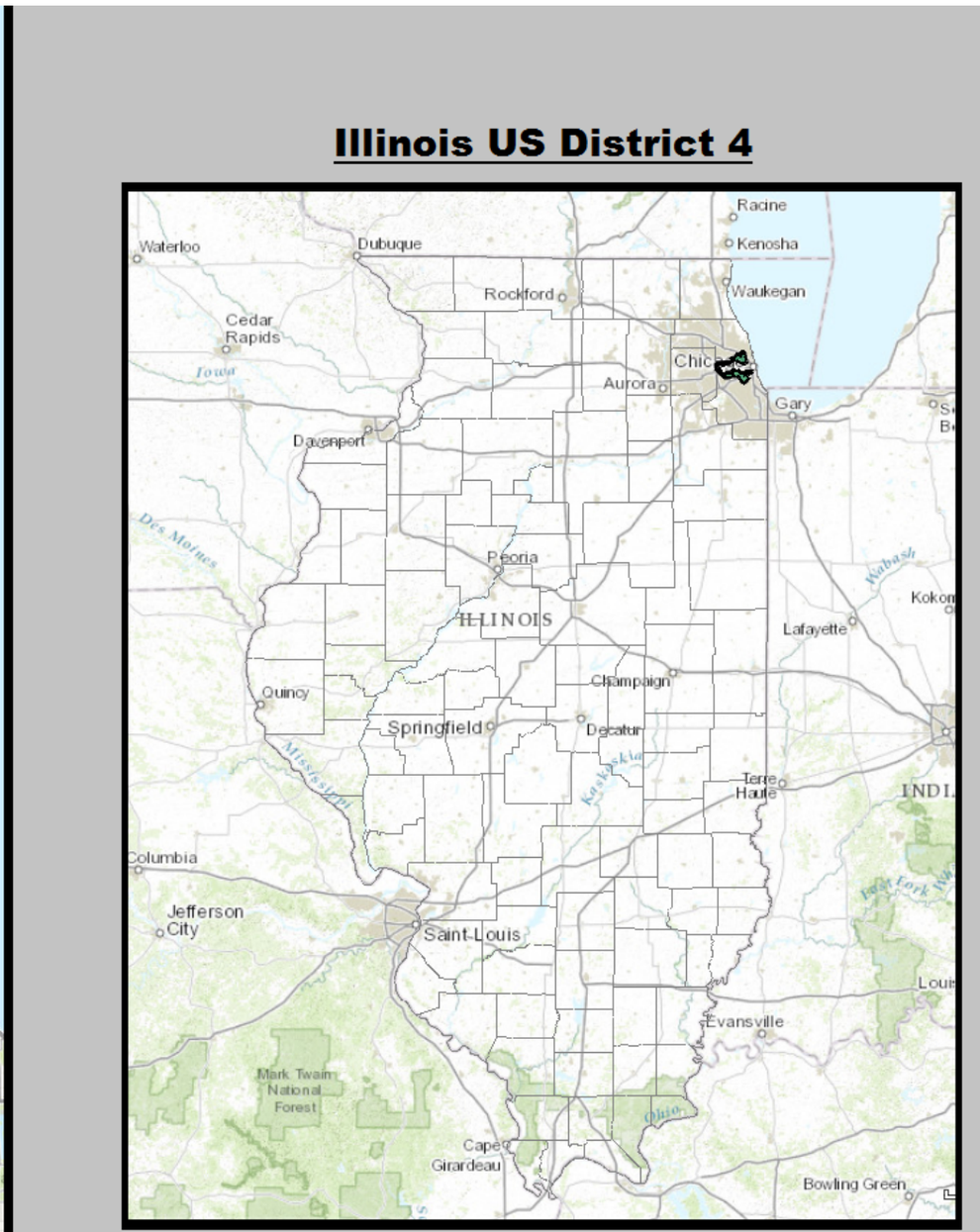
North Carolina US District 12



US Congressional districts since 2013
Source: http://nationalatlas.gov, 1 Million Scale project.

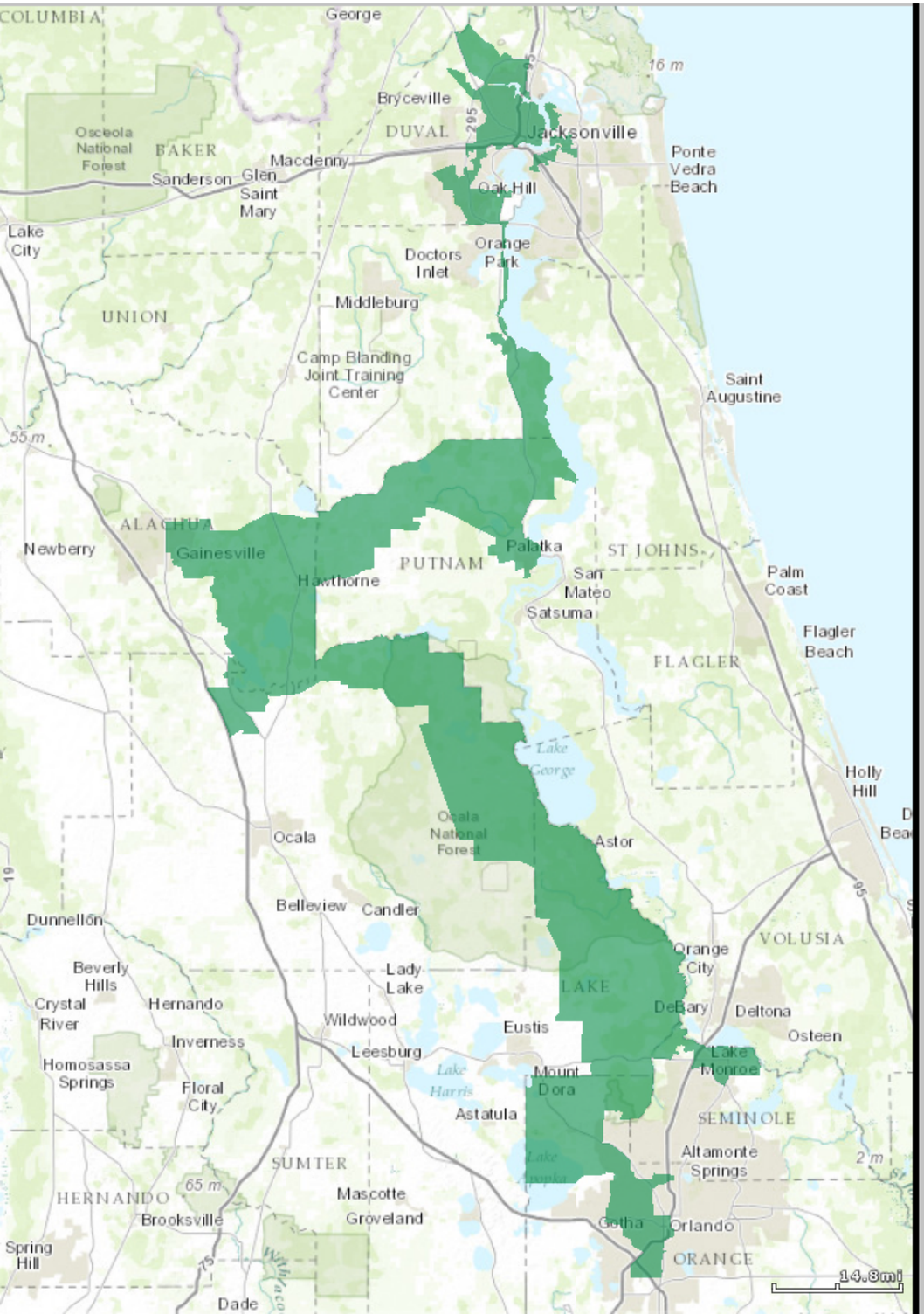
Examples: Illinois' 4th District

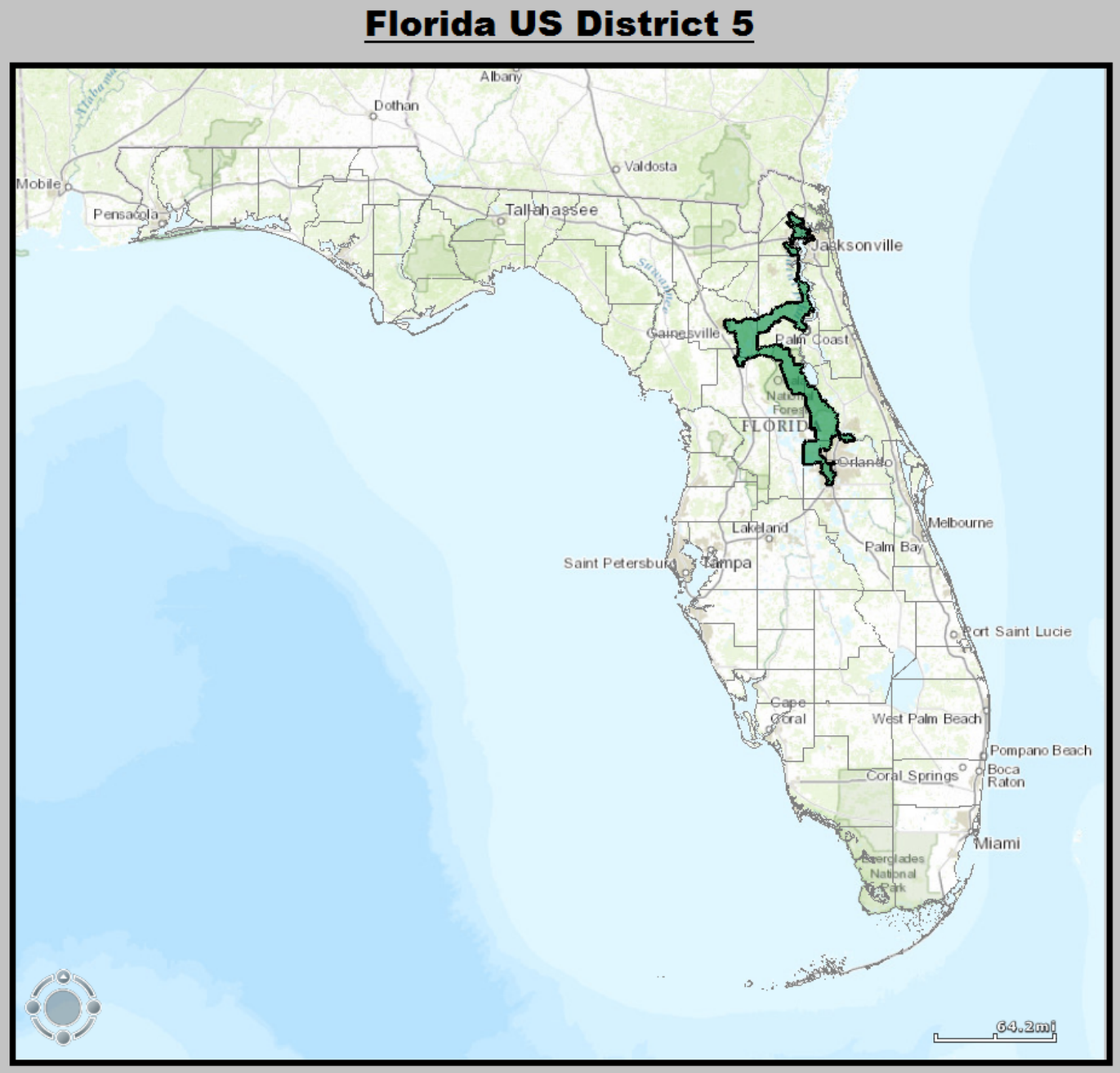




US Congressional districts since 2013
Source: http://nationalatlas.gov, 1 Million Scale project.

Examples: Florida's 5th District





US Congressional districts since 2013
Source: http://nationalatlas.gov, One-Million-Scale project.

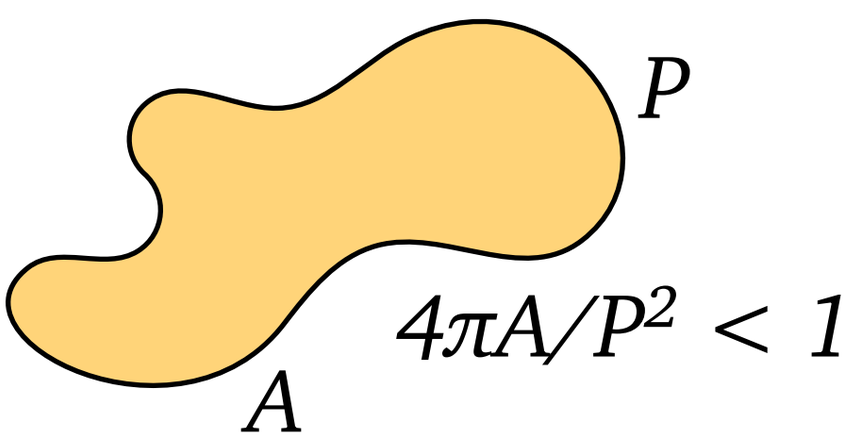
Measures of gerrymandering

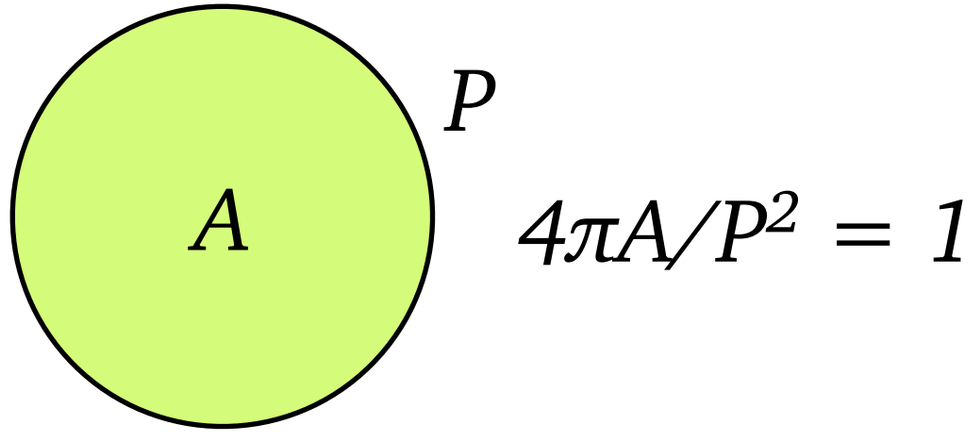
- A common description of gerrymandering is “you know it when you see it”.
- But, can we tell *mathematically* if a district is gerrymandered? Such a method would be more objective, and less susceptible to bias.
- There are several competing methods, usually described as **compactness measures**, as they attempt to give a precise meaning to the word “compactness” in the context of congressional districts.
- Each method we present here is, essentially, an answer to the question “how much does the given district differ from an *ideal* district?”
- The methods differ in their understanding of what an “ideal district” is, and how we measure the difference between that and the given district.

The Polsby—Popper ratio

- The first method begins with the assumption that an ideal district should be a circle. To understand it, we need an important result from geometry:

Theorem (The Isoperimetric Inequality): *For any "simple closed curve" in the plane with perimeter P (in ft, say), that bounds an area A (in ft^2), we have that $4\pi A \leq P^2$. Equivalently, $4\pi A/P^2 \leq 1$.*





- Since $4\pi A = P^2$ when the closed curve is a circle, it follows that amongst all curves with the same perimeter, the circle bounds the maximum possible area.

- The difference between the ratio $4\pi A/P^2$ and 1 is a measure of how much the area enclosed by the curve differs from that of a circle with the *same perimeter*.

The Polsby—Popper ratio (cont'd)

- The **Polsby—Popper ratio** (named for lawyers Daniel Poslby and Robert Popper) for a given congressional district, with perimeter P and area A, is exactly the ratio $4\pi A/P^2$.

- The intent is that a district with a higher (i.e., closer to 1) ratio is *less* gerrymandered, while one with a lower ratio is *more* gerrymandered.

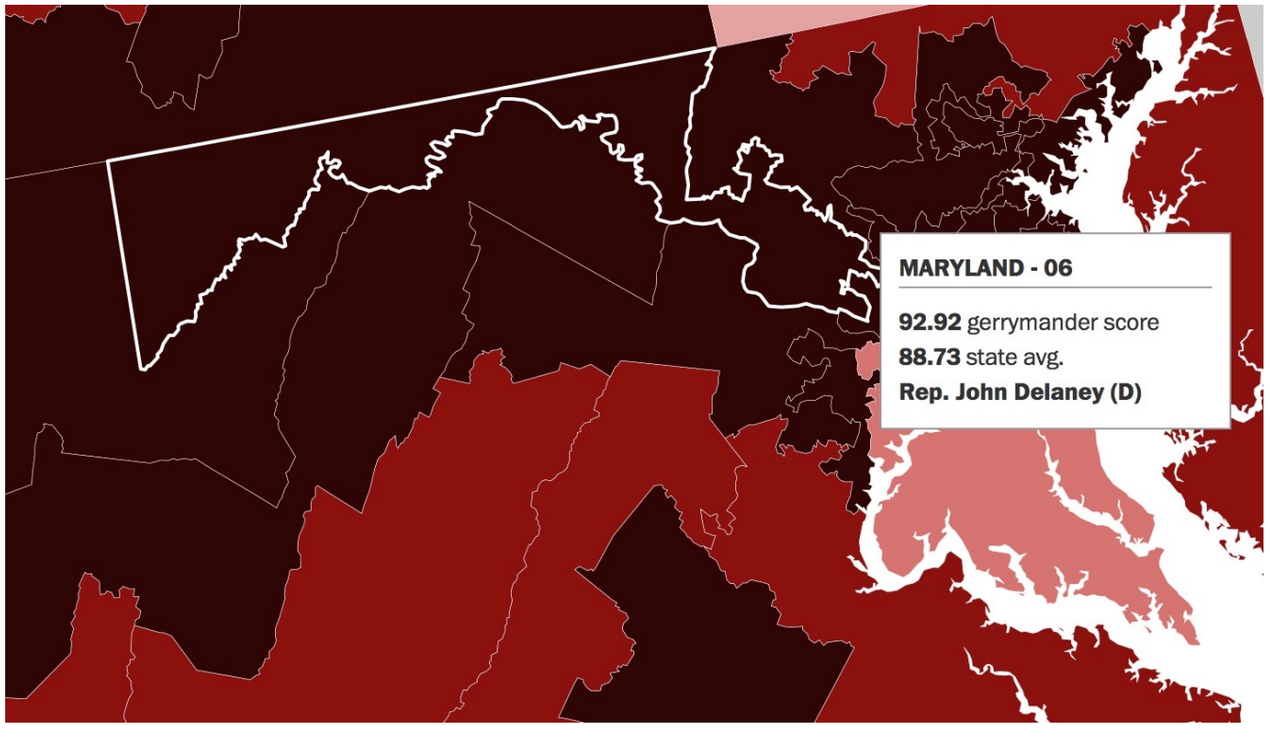
- A major advantage of this method is that it is extremely easy to determine, using publicly available data.

- In fact, the Washington Post has done it for us (their numbers are the result of multiplying $1\text{-}4\pi A/P^2$ by 100 to obtain an "index"; to obtain the original PP ratios, just divide their "gerrymander score" by 100, and subtract the result from 1).

The Polsby—Popper ratio (cont'd)

- Can you think of some potential problems with this measure?
  - Squares and rectangles, which don't seem gerrymandered, don't get "perfect" scores.
  - The boundaries of states (which districts must respect), as well as natural boundaries (rivers, lakes, etc) can cause reasonable districts to appear gerrymandered by this measure. For example, consider

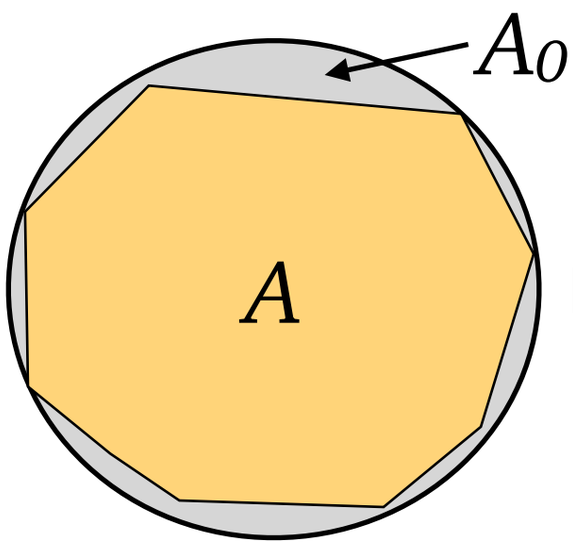
Maryland's 6th Congressional District:



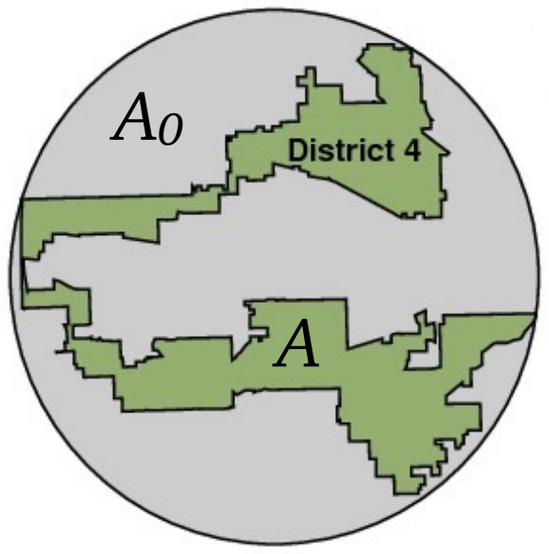
- This district has a very small PP ratio of 0.071.
- But, most of the strange, jagged southern boundary of this district is the Maryland/ West Virginia border, formed by the Potomac River.

The Reock ratio

- This method also begins with the assumption that an ideal district should be a circle, but identifies a different circle as being ideal.
- The **Reock ratio** is the ratio A/A_0, where A is the area of the district, and A_0 is the area of the smallest circle containing the district.
- Again, the intent is that high ratios (i.e., closer to 1) are *less* gerrymandered, while low ratios are *more* gerrymandered.



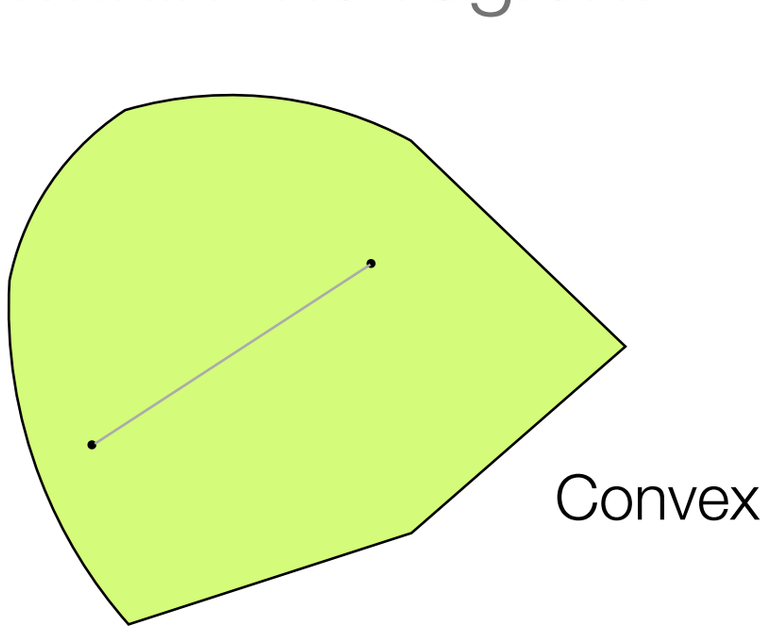
High Reock ratio



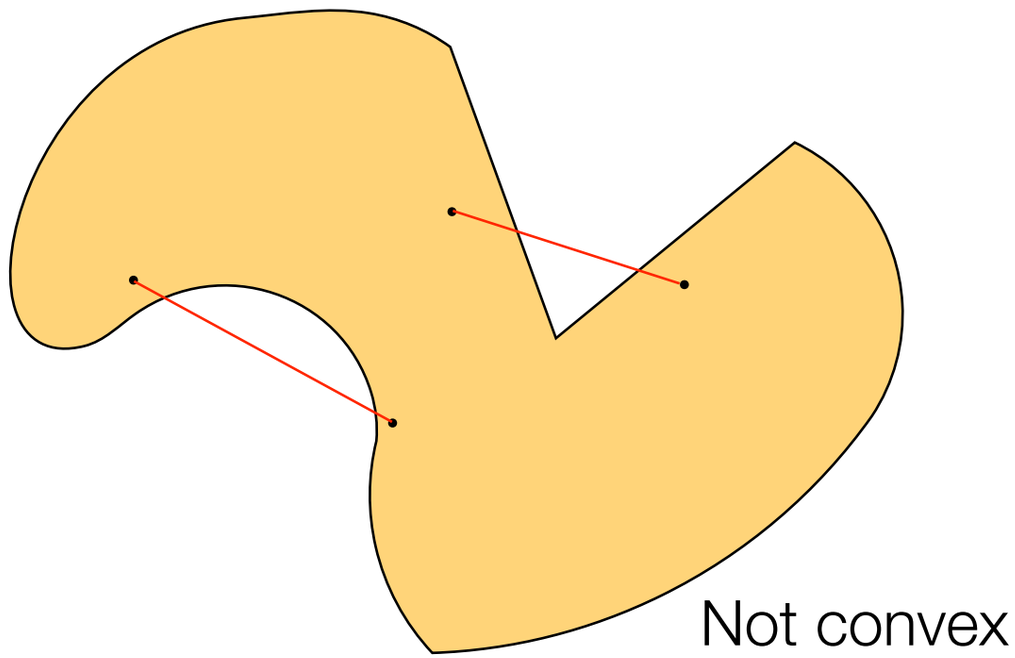
Low Reock ratio
(Illinois 4th district)

The convex hull ratio

- A region is **convex** if whenever two points in the region are connected by a straight line, that line lies entirely within the region.



Convex



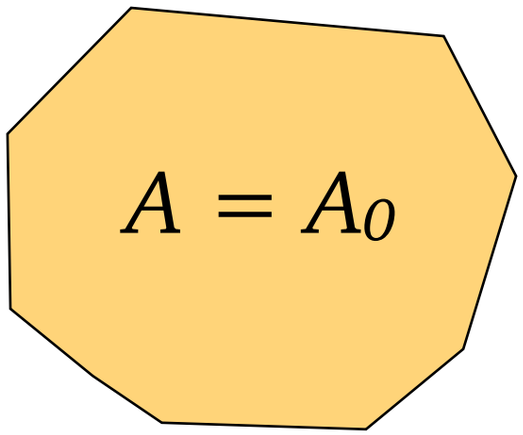
Not convex

- In particular, both rectangles (and all regular polygons) and circles are convex.

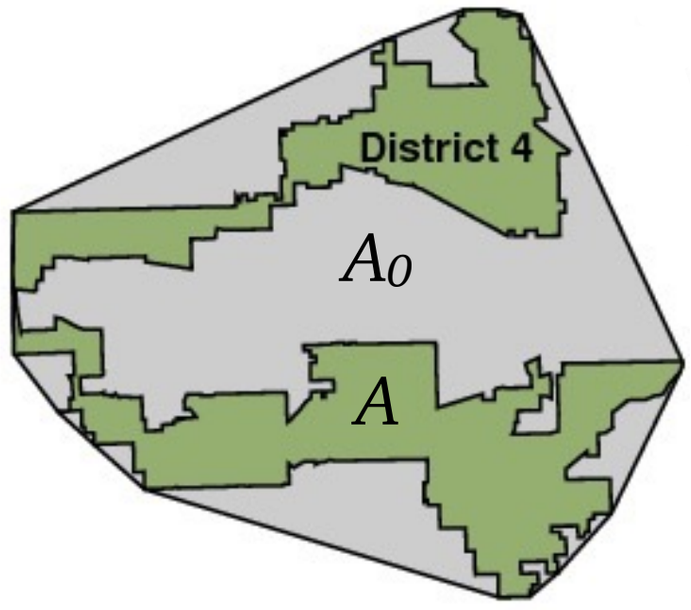
The convex hull ratio (cont'd)

- This method begins with the assumption that an ideal district should be convex.

- The **convex hull ratio** is the ratio A/A_0, where A is the area of the district, and A_0 is the area of the smallest convex region (the **convex hull**) containing the district.



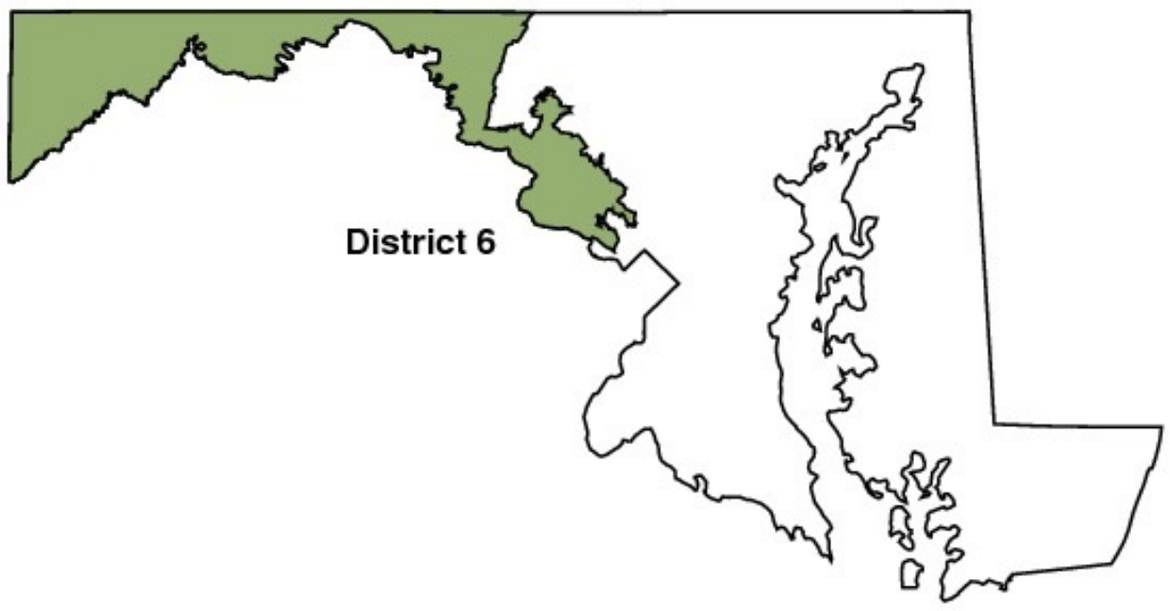
High (perfect!) convex hull ratio



Low convex hull ratio (Illinois 4th district)

Area ratios

- All of the previous measures are examples of **area ratios**, and each is subject to some of the same issues as the Polsby—Popper ratio, namely they do not take into account state and natural boundaries.
- In particular, each gives relatively poor scores to Maryland's 6th Congressional district:
  - Polsby—Popper: 0.071
  - Reock: 0.121
  - Convex hull: 0.562



Bizarreness

- In 2007, economists Christopher Chambers and Alan Miller introduced an alternate measure which addresses some of the difficulties with the area measures we've seen.

- The **bizarreness** of a district is (essentially) the probability that the shortest path *within the state* between two people in the district stays within the district. (For more details, see their paper linked above, or the more elementary explanation in this AMS Feature Column on Congressional Redistricting.)

- While grammatically unfortunate, the intent is that high bizarreness (i.e., close to 1) means a district is less gerrymandered.

- Convex districts still get a measure of 1, but districts whose non-convexity is due to state boarders have bizarreness close to 1 as well.

- In particular, they compute the bizarreness of Maryland's 6th district to be a relatively mild 0.926, but that of Maryland's 3rd district to be an egregious 0.140.

- Optional (but recommended!) reading:
  - Dave Austin's AMS Feature Column "Congressional Redistricting and Gerrymandering" (from which much of the diagrams and data for these slides have been taken)
  - Vox's "What is gerrymandering?"
  - The Washington Post's recent Wonkblog entries on gerrymandering: 1, 2
  - Hodge, Marshall, Patterson, "Gerrymandering and convexity", College Math. J., 2010.

- Problem set 5 is due tomorrow, in class.