The following is a version of Theorem 15.11 in Robinson & Ullman. I found the proof in the text a bit "lacking", so I've given my own below.

Theorem 1 (von Neumann's Min-Max Theorem)**.** *In a two-person, zero-sum game:*

(i) *Every Nash equilibrium (P, Q) is a doubly prudent mixed outcome, and $\overline{r} = \overline{c}$. In particular, prudent mixed strategies exist for both players.*

(ii) *Conversely, every doubly prudent mixed outcome (P, Q) is a Nash equilibrium.*

Proof: (i): Suppose that (P, Q) is a Nash equilibrium. Since P is a best response to Q, $E(P, Q)$ is the guarantee of Q. Similarly, since Q is a best response to P, $E(P, Q)$ is also the guarantee of P.

Towards a contradiction, suppose that P was *not* a prudent mixed strategy. Then, there must be another mixed strategy P' having a *better* guarantee for Row, i.e., $E(P', Q') > E(P, Q)$, where Q' is a best response (for Column) to P'. Thus,

$$E(P', Q') > E(P, Q) \geq E(P', Q) \geq E(P', Q')$$

where the second inequality is from the fact that P is a best response to Q, and the third inequality is from the fact that Q' is a best response to P'. But this shows that $E(P', Q') > E(P', Q')$, which is a contradiction, as no number can be strictly larger than itself. Thus, P is a prudent mixed strategy, and $\overline{r} = E(P, Q)$.

The argument for Q is similar, we give it for completeness: Towards a contradiction, suppose that Q was *not* a prudent mixed strategy. Then, there must be another mixed strategy Q' having a *better* guarantee for Column, i.e., $E(P', Q') < E(P, Q)$, where P' is a best response (for Row) to Q'. Thus,

$$E(P', Q') < E(P, Q) \leq E(P, Q') \leq E(P', Q')$$

where the second inequality is from the fact that Q is a best response to P, and the third inequality is from the fact that P' is a best response to Q'. But this shows that $E(P', Q') < E(P', Q')$, again a contradiction. Thus, Q is a prudent mixed strategy, and $\overline{c} = E(P, Q) = \overline{r}$.

(ii) Conversely, suppose that (P, Q) is a doubly prudent mixed outcome. Since P is prudent, and $\overline{r}$ is the guarantee of a prudent mixed strategy, we have that $\overline{r} \leq E(P, Q)$. Similarly, since Q is prudent, $E(P, Q) \leq \overline{c}$. But, we've seen in part (i) that $\overline{r} = \overline{c}$ in any such game, so it follows that $\overline{r} = E(P, Q) = \overline{c}$.

Since P is prudent, all payoffs against P must be at least $\overline{r}$, that is, $E(P, Q) = \overline{r} \leq E(P, S)$ for all strategies S for Column. Likewise, since Q is prudent, all payoffs against Q must be at most $\overline{c}$, that is, $E(P, Q) = \overline{c} \geq E(R, Q)$ for all strategies R for Row. But combined, these facts show that (P, Q) is a Nash equilibrium (c.f., the inequalities near the bottom of p. 275 of R&U). $\square$