Vector-valued functions (and a bit more)

Name: _____

Math 1920 - Sections 221 and 222 - TA: Itamar Oliveira

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Problems

(1) State the type of quadric surface and describe the trace obtained by intersecting with the given plane.

(a) $x^2 + (\frac{y}{4})^2 + z^2 = 1$, $y = 0$.

(b) $(\frac{x}{3})^2 + (\frac{y}{5})^2 - 5z^2 = 1$, $x = 0$.

(2) Find an equation of the form $r = f(\theta, z)$ in cylindrical coordinates for the following surfaces.

(a) $z = x + y$.

(b) $z = 3xy$.

(3) Find an equation of the form $\rho = f(\theta, \phi)$ in spherical coordinates for the following surfaces.

(a) $z^2 = 3(x^2 + y^2)$.

(b) $x^2 - y^2 = 4$.

(4) Use sine and cosine to parametrize the intersection of the surfaces $x^2 + y^2 = 1$ and $z = 4x^2$.

(5) Let $\mathbf{r}(t) = \langle 3\cos t, 5\sin t, 4\cos t\rangle$. Show that $\|\mathbf{r}(t)\|$ is constant and use this to conclude that $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ are orthogonal.

(6) A fighter plane, which can shoot a laser beam straight ahead, travels along the path

$$\mathbf{r}(t) = \langle t - t^3, 12 - t^2, 3 - t\rangle.$$

Show that the pilot cannot hit any target on the x-axis.