PRELIM REVIEW

Math 1920 - Sections 221 and 222 - TA: Itamar Oliveira

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PROBLEMS

(1) Find the equation of the plane that contains the lines $\mathbf{r}_1(t) = \langle t, 2t, 3t \rangle$ and $\mathbf{r}_2(t) = \langle 3t, t, 8t \rangle$.

SOLUTION: $13x + y - 5z = 0$

(2) Sketch the set described in cylindrical coordinates.

(a) $r = 4$.

SOLUTION: Cilinder of radius 4.

(b) $\theta = \frac{\pi}{3}$.

SOLUTION: Half-plane that projects onto the ray $\theta = \pi/3$

(c) $z^2 + r^2 \leq 4$.

SOLUTION: Sphere of radius 2.

(3) Evaluate the limit or determine that it does not exist.

(a) $\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2+y^2}}$.

SOLUTION: Since, $0 \leq |y| \leq \sqrt{x^2+y^2}$, the limit is zero by the Squeeze theorem.

(b) $\lim_{(x,y)\to(0,0)} \frac{|x|}{|x|+|y|}$.

SOLUTION: Try lines $y = mx$ to show that the limit does not exist.

(c) $\lim_{(x,y)\to(0,2)} (1+x)^{y/x}$.

SOLUTION: Set $f(x,y) = (1+x)^{y/x}$ and show that $\lim_{(x,y)\to(0,2)} \ln f(x,y) = 2$. By continuity of the exponential, the required limit is e^2.

(d) $\lim_{(x,y)\to(0,0)} \frac{x^2-y^2}{\sqrt{x^2+y^2}}$.

SOLUTION: The limit is 0. Use polar coordinates and the Squeeze Theorem.

(4) Suppose that the plane tangent to $z = f(x,y)$ at $(-2,3,4)$ has equation $4x + 2y + z = 2$. Estimate $f(-2.1, 3.1)$.

SOLUTION: $f(-2.1, 3.1) \approx 4 - 4(-0.1) - 2(0.1) = 4.2$.

(5) A fighter plane, which can shoot a laser beam straight ahead, travels along the path

$$\mathbf{r}(t) = \langle t - t^3, 12 - t^2, 3 - t \rangle.$$

Show that the pilot cannot hit any target on the x-axis.

(6) Find a vector normal to the surface $3z^3 + x^2y - y^2x = 1$ at $P = (1,-1,1)$.

SOLUTION: $\langle -3, 3, 9 \rangle$.