Prelim 2 Review

Math 1920 - Sections 221 and 222 - TA: Itamar Oliveira

Name: Solutions

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Problems

(1) Find the critical points of $f(x,y) = x^3 - xy + y^3$. Then, if possible, use the Second Derivative Test to determine if they are local maxima, local minima or saddle points.

Solution: The critical points are $(0,0)$ (saddle point) and $(\frac{1}{3}, \frac{1}{3})$ (local minimum).

(2) Find the minimum and maximum values of the function f subject to the given constraint.

$$f(x,y) = 2x + 3y; \quad x^2 + y^2 = 4.$$

Solution: The minimum is $\frac{-26}{\sqrt{13}}$ and the maximum is $\frac{26}{\sqrt{13}}$.

(3) Use spherical coordinates to calculate the triple integral of $f(x,y,z) = \sqrt{x^2+y^2+z^2}$ over $W = \{(x,y,z) : x^2+y^2+z^2 \leq 2z\}$.

Solution: $\frac{8\pi}{5}$.

(4) Compute $\iint_{\mathcal{D}} (x+3y)\mathrm{d}x\mathrm{d}y$, where $\mathcal{D}$ is the shaded region in the figure below. *Hint:* Use the map $\Phi(u,v) = (u-2v, v)$.

Solution: 80.

(5) Let $\mathcal{C} = \mathcal{C}_1 + \mathcal{C}_2$ where $\mathcal{C}_1$ is the quarter circle $x^2+y^2 = 4$, $z = 0$, from $(0,2,0)$ to $(2,0,0)$, and where $\mathcal{C}_2$ is the line segment from $(2,0,0)$ to $(3,3,3)$. Compute the work done along $\mathcal{C}$ by the force $\mathbf{F}(x,y,z) = \langle -y+z, z-x, x+y+z \rangle$.

Solution: This vector field is conservative, $f(x,y,z) = xz - xy + zy + \frac{z^2}{2}$ is a potential function, so the work is $f(3,3,3) - f(0,2,0) = \frac{27}{2}$.

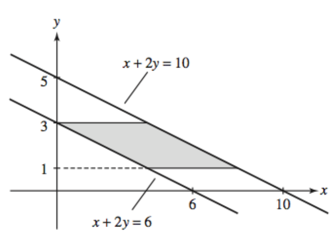

Figure 1: Problem 4