

PROBLEMS

- (1) Find the critical points of $f(x, y) = x^3 - xy + y^3$. Then, if possible, use the Second Derivative Test to determine if they are local maxima, local minima or saddle points.
- (2) Find the minimum and maximum values of the function f subject to the given constraint.

$$f(x, y) = 2x + 3y; \quad x^2 + y^2 = 4.$$

- (3) Use spherical coordinates to calculate the triple integral of $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ over $W = \{(x, y, z) : x^2 + y^2 + z^2 \leq 2z\}$.
- (4) Compute $\iint_{\mathcal{D}} (x + 3y) dx dy$, where \mathcal{D} is the shaded region in the figure below. *Hint:* Use the map $\Phi(u, v) = (u - 2v, v)$.
- (5) Let $\mathcal{C} = \mathcal{C}_1 + \mathcal{C}_2$ where \mathcal{C}_1 is the quarter circle $x^2 + y^2 = 4$, $z = 0$, from $(0, 2, 0)$ to $(2, 0, 0)$, and where \mathcal{C}_2 is the line segment from $(2, 0, 0)$ to $(3, 3, 3)$. Compute the work done along \mathcal{C} by the force $\mathbf{F}(x, y, z) = \langle -y + z, z - x, x + y + z \rangle$.

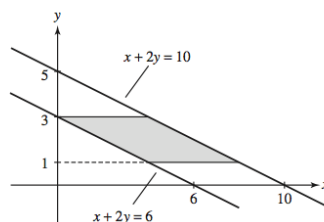


Figure 1: Problem 4