

CONSERVATIVE VECTOR FIELDS

Math 1920 - Sections 221 and 222 - TA: Itamar Oliveira

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REVIEW

- (1) A vector field \mathbf{F} on a domain \mathcal{D} is called *path-independent* if for any two points $P, Q \in \mathcal{D}$, we have

$$\int_{C_1} \mathbf{F} \cdot d\mathbf{r} = \int_{C_2} \mathbf{F} \cdot d\mathbf{r}$$

for any two paths C_1 and C_2 in \mathcal{D} from P to Q .

- (2) The Fundamental Theorem for Conservative Vector Fields: If $\mathbf{F} = \nabla f$, then

$$\int_C \mathbf{F} \cdot d\mathbf{r} = f(Q) - f(P)$$

for any path \mathbf{r} from P to Q in the domain of \mathbf{F} . This shows that conservative vector fields are path independent. The converse is also true: on an open, connected domain, a path-independent vector field is conservative.

- (3) The work W exerted on an object along a curve \mathcal{C} is given by:

$$W = \int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}.$$

The work performed *against* \mathbf{F} is the quantity $-\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$.

PROBLEMS

(1) Evaluate

$$\oint_{\mathcal{C}} \sin x \, dx + z \cos y \, dy + \sin y \, dz$$

where \mathcal{C} is the ellipse $4x^2 + 9y^2 = 36$, oriented clockwise.

(2) Determine whether the vector field in the picture below is conservative or not.

(3) Let $\mathbf{F}(x, y) = \left\langle \frac{1}{x}, \frac{-1}{y} \right\rangle$. Show that the work against \mathbf{F} required to move an object from $(1, 1)$ to $(3, 4)$ is the same no matter what path we pick in the first quadrant. Compute this work.

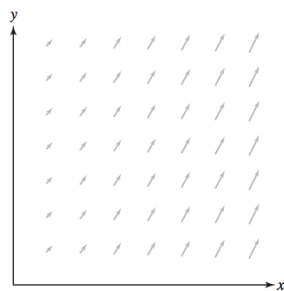


Figure 1: Problem 2