IMPLICIT DIFFERENTIATION   NAME: _____

Math 1110 - Instructor: Itamar Oliveira   March 2, 2020

1   INTRODUCTION AND EXAMPLES

The functions that we have met so far can be described by expressing one variable **explicitly** in terms of another variable, for example

$$y = \sqrt{x^3 + 1} \quad \text{or} \quad y = x \sin x.$$

Some functions, however, are defined implicitly by a relation between x and y such as

$$x^3 + y^3 = 6xy.$$

One can not solve the equation above to find $y = f(x)$ (google *folium of Descartes* and look at its picture to understand why). Fortunately, we don't need to solve it for y in order to find y'! We can use the method of **implicit differentiation**.

1. (a) If $x^2 + y^2 = 25$, find $\dfrac{dy}{dx}$.

    (b) Find an equation of the tangent line to the circle $x^2 + y^2 = 25$ at the point $(3, 4)$.

2. (a) Find y' if $x^3 + y^3 = 6xy$.

    (b) Find the tangent to the folium of Descartes $x^3 + y^3 = 6xy$ at the point $(3, 3)$.

    (c) At what point in the first quadrant is the tangent line horizontal?

3. Find the two points where the curve $x^2 + xy + y^2 = 7$ crosses the x-axis, and show that the tangents to the curve at these points are parallel. What is the common slope of these tangents?

4. Below are some curves $x^{\frac{p}{q}} + y^{\frac{p}{q}} = 1$, where p is even and q is odd. These curves are sometimes called *astroids* when $p/q < 1$. At what point(s) is the slope of the tangent line equal to 1 or -1 if $p/q = \frac{4}{3}$? How about if $p/q = \frac{2}{5}$?