Math 2940: Prelim 2

July 26, 2016

Name:

NetID:

Instructions: This exam is 75 minutes long. It has 11 pages including the cover and 6 questions worth a total of 100 points. No written or electronic aids are allowed.

Please fully explain all your answers except when explicitly instructed otherwise. If you need more space to answer a question, use the back of the page or the blank sheet at the end of the exam. Label your work clearly if you use extra space.

Academic integrity is expected of all Cornell University students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare I shall not give, use, or receive unauthorized aid in this examination.

Please sign below to indicate that you have read and agree to these instructions.

Signature:

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Total score:	

1. (10 points) Let

$$A = \begin{bmatrix} 3 & 8 & -2 & 1 & 1 \\ -2 & 0 & 4 & 1 & 5 \end{bmatrix}.$$

Find the dimensions of Col(A), Row(A), and Nul(A). (It's possible to do this without any row operations, but you must explain your reasoning.)

2. (16 points) Let W be the set of all vectors $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \in \mathbf{R}^4$ such that $x_1 = 5x_3$ and $x_1 + x_2 = x_4$.

(a) (6 points) Prove that W is a subspace of $\mathbf{R}^4$.

(b) (10 points) Find a basis for W.

3. (22 points) Let A be a 3×3 matrix whose characteristic polynomial is $(4-\lambda)(2-\lambda)^2$.

(a) (6 points) Is A invertible? Explain.

(b) (8 points) What is $\mathrm{Nul}(A-I)$? What is $\det(A-I)$?

(c) (8 points) Suppose that

$$A\begin{bmatrix}1\\3\\-3\end{bmatrix}=\begin{bmatrix}2\\6\\-6\end{bmatrix}, \quad A\begin{bmatrix}0\\4\\1\end{bmatrix}=\begin{bmatrix}0\\8\\2\end{bmatrix}, \quad A\begin{bmatrix}1\\7\\1\end{bmatrix}=\begin{bmatrix}4\\28\\4\end{bmatrix}.$$

Find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

4. (14 points) Let

$$\mathcal{B} = \left\{ \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix} \right\}$$

be a basis for $\mathbf{R}^3$, and let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be a linear transformation whose matrix with respect to $\mathcal{B}$ is

$$[T]_{\mathcal{B}} = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & -2 \\ -1 & 3 & -4 \end{bmatrix}.$$

(a) (7 points) Let

$$\mathbf{x} = \begin{bmatrix} 4 \\ -1 \\ 4 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ -5 \\ 3 \end{bmatrix}.$$

Compute $[\mathbf{x}]_{\mathcal{B}}$ and $[T(\mathbf{x})]_{\mathcal{B}}$.

(b) (7 points) Compute $T\left(\begin{bmatrix} -2 \\ 2 \\ 0 \end{bmatrix}\right)$.

5. (18 points) Let

$$A = \begin{bmatrix} 0.68 & 0.08 \\ 0.12 & 0.72 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} 0.8 & 0 \\ 0 & 0.6 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}^{-1}$$

and consider the discrete dynamical system $\mathbf{x}_{k+1} = A\mathbf{x}_k$.

(a) (10 points) If $\mathbf{x}_0 = \begin{bmatrix} -6 \\ 1 \end{bmatrix}$, find a formula for $\mathbf{x}_k$.

(b) (8 points) Is the origin a saddle point, attractor, or repeller for this dynamical system? Draw a graphical description of the system, indicating the directions of greatest attraction/repulsion and showing several typical trajectories.

6. (20 points) Let A be a 4×5 matrix whose columns are $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \mathbf{a}_4, \mathbf{a}_5$ in order. Suppose that $5\mathbf{a}_1 - 3\mathbf{a}_2 + \mathbf{a}_4 = \mathbf{0}$, and that the equation $A\mathbf{x} = \mathbf{b}$ has at least one solution for every $\mathbf{b} \in \mathbf{R}^4$.

(a) (7 points) Find the rank of A and the dimension of $\mathrm{Nul}(A)$.

(b) (5 points) Find a nonzero vector in $\mathrm{Nul}(A)$. What is a basis for $\mathrm{Nul}(A)$?

(c) (8 points) What is $\mathrm{rref}(A)$? (*Hint:* The columns of $\mathrm{rref}(A)$ satisfy the same relations as the columns of A.)

(This page for scratch work.)