Math 4220: Hint for 2.5.20 and extra credit problem

For exercise 2.5.20, the first step is to find $g(x, y)$ such that $\frac{\partial g}{\partial y} = \frac{\partial u}{\partial x}$. Then, look at the function $h(x, y) = \frac{\partial g}{\partial x} + \frac{\partial u}{\partial y}$. What is $\frac{\partial h}{\partial y}$, and what does that imply about h?

If the various functions and partial derivatives are confusing, try working with a concrete example to keep yourself anchored.

Extra credit problem: Exercise 2.5.20 specifies that the domain of u is a disk. What's more, exercise 2.5.21 (not assigned) shows that when the domain of u is the punctured plane $\mathbf{R}^2 - \{(0, 0)\}$, the "universal procedure" to find a harmonic conjugate can fail. Explain where in your answer for exercise 2.5.20 you used the assumption that the domain of u is a disk, and why that part of the argument fails for the punctured plane.