Math 4220: Prelim 2

November 5, 2015

Name:

Instructions: This exam is 75 minutes long. It has 7 pages including the cover and 5 questions worth a total of 100 points. No written or electronic aids are allowed.

Please fully explain all your answers. If you need more space to answer a question, use the back of the *preceding* sheet or the blank sheet at the end of the exam. Label your work clearly if you use extra space.

Academic integrity is expected of all Cornell University students at all times, whether in the presence or absence of members of the faculty. Understanding this, I declare I shall not give, use, or receive unauthorized aid in this examination.

Please sign below to indicate that you have read and agree to these instructions.

Signature:

1:	
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Total score:	

1. (20 points) Suppose the functions f and g are both analytic on a domain that contains the closed unit disk $\{|z| \leq 1\}$. If $f(z) = g(z)$ for all z with $|z| = 1$, prove that $f(z) = g(z)$ for all z with $|z| < 1$.

2. (20 points) Find the Laurent series for $f(z) = \dfrac{5}{z} + \dfrac{9}{z-4}$ centered at 1 that converges on the annulus $\{1 < |z-1| < 3\}$.

Hint: Write everything in terms of $w = z - 1$.

3. (20 points) Let $f(z) = \dfrac{a}{z} + \dfrac{b}{z-1} + \dfrac{c}{z-2}$, where $a, b, c \in \mathbf{C}$ are constants. Let $[0,2]$ denote the line segment $\{x + iy \in \mathbf{C} : 0 \leq x \leq 2,\ y = 0\}$, and let $D = \mathbf{C} \setminus [0,2]$. For which values of a, b, c does f have an antiderivative on D? Prove your answer.

4. (20 points) Classify the singularity at 0 of $f(z) = \dfrac{z\sin(z)}{\sin(z) - z}$ as essential, removable, or a pole (whose order you must specify). Also find the residue of f at 0.

5. (20 points) Using residue theory, compute p.v. $\displaystyle\int_{-\infty}^{\infty} e^{-ix} \frac{x}{x^2+4} dx$.

(This page for scratch work.)