Math 4740 **Name:** _____
Spring 2013
Prelim
3/1/13
Time Limit: 50 Minutes

This exam contains 5 pages (including this cover page) and 3 problems. Check to see if any pages are missing.

You may *not* use books, notes, calculator, phone, tablet, laptop or any other device on this exam.

- **Explain your answers**. A correct answer with no explanation may receive no credit.
- **Organize your work** in a reasonably neat and coherent way. Work scattered all over the page without a clear ordering will receive very little credit.
- **If you use a theorem from class you should indicate this** and explain why the theorem may be applied.
- **If you need more space, use the back of the pages**; clearly indicate when you have done this.

Problem	Points	Score
1	20	
2	25	
3	15	
Total:	60	

1. (20 points) Let X_n be a Markov chain with states a, b, c, d, e and transition matrix

$$p = \begin{array}{c|ccccc} & a & b & c & d & e \\ a & 1 & 0 & 0 & 0 & 0 \\ b & .5 & 0 & .5 & 0 & 0 \\ c & 0 & .5 & 0 & .5 & 0 \\ d & 0 & 0 & 0 & .8 & .2 \\ e & 0 & 0 & 0 & .6 & .4 \end{array}$$

(a) (3 points) List all of the irreducible closed sets.

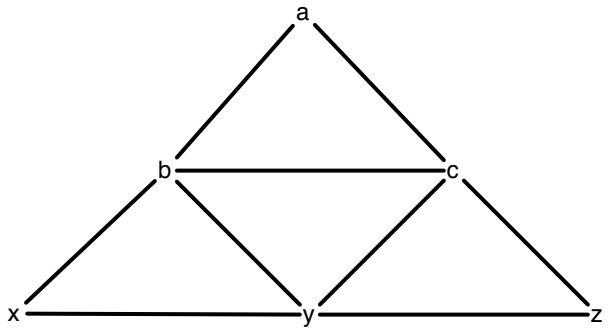
(b) (2 points) Which states are recurrent?

(c) (5 points) Find $\lim_{n\to\infty} p^n(d, e)$.

(d) (5 points) Find $P_c(T_a < T_e)$.

(e) (5 points) Find $\lim_{n\to\infty} p^n(c, e)$.

2. (25 points) Let X_n be random walk on the following graph

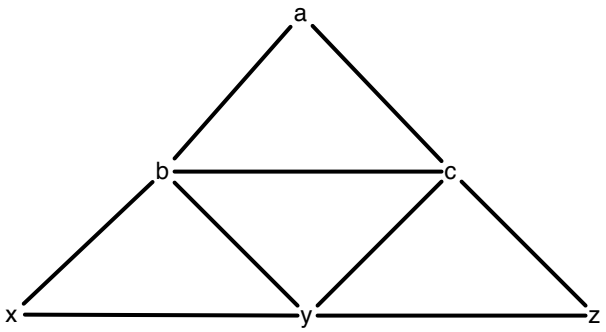


and let $T_a = \min\{n \geq 1 : X_n = a\}$ be the time of first return to a.

(a) (5 points) Find $P_a(T_a = 3)$.

(b) (5 points) Find $\pi(a)$.

(c) (2 points) Find $E_a T_a$.



Let $T_B = \min\{n \geq 1 : X_n \in B\}$ where $B = \{x, y, z\}$.

(d) (5 points) Find $E_a T_B$.

(e) (5 points) Starting at 11pm an ant performs random walk on the above anthill at a rate of one step every 10 seconds. Whenever she is at vertex a she picks up 5 grains of sugar, and whenever she is anywhere on the bottom row B if she has any sugar then she drops off 1 grain. Estimate how much sugar she is carrying at midnight.

(f) (3 points) Explain the source(s) of error in your estimate for part e.

3. (15 points) Let X_n be an **irreducible** Markov chain on a finite state space S with transition matrix p and stationary distribution π.

   For each sentence below circle True or False, and **briefly explain** your answer.

   (a) All states are recurrent. True False

   (b) There exists $n \geq 1$ such that all entries of the matrix p^n are positive. True False

   (c) If $p(x, x) > 0$ for all $x \in S$, then $\lim_{n\to\infty} p^n(x, y) = \pi(y)$ for all $x, y \in S$. True False

   (d) If μ is a row vector such that $\mu p = \mu$, then $\mu = \pi$. True False

   (e) If $\sum_{x\in S} p(x, y) = 1$ for all $y \in S$, then $\pi(x) = \pi(y)$ for all $x, y \in S$. True False