# MATH 4740 HW 10 Solution.

## Textbook Exercises:

6.5 (a) Denote the probabilities of winning, drawing and losing by $P_1$, $P_2$ and $P_3$ respectively. Then under neutral risk, we will have

$$\begin{cases} (120-100)P_1 + (110-100)P_2 + (84-100)P_3 = 0 \\ (30-50)P_1 + (55-50)P_2 + (60-50)P_3 = 0 \\ P_1 + P_2 + P_3 = 1 \end{cases}$$

$$\Rightarrow \begin{cases} P_1^* = 0.3 \\ P_2^* = 0.2 \\ P_3^* = 0.5 \end{cases}$$

So the option price to buy Netscape for \$50 is

$$C = P_1^* \cdot 0 + P_2^* \cdot 5 + P_3^* \cdot 10 = \$6$$

(b) Denote the amount of cash, the shares of Microsoft Stock to buy/sell, and the shares of Netscape stock to buy/sell by $x$, $y$, $z$ respectively. Then

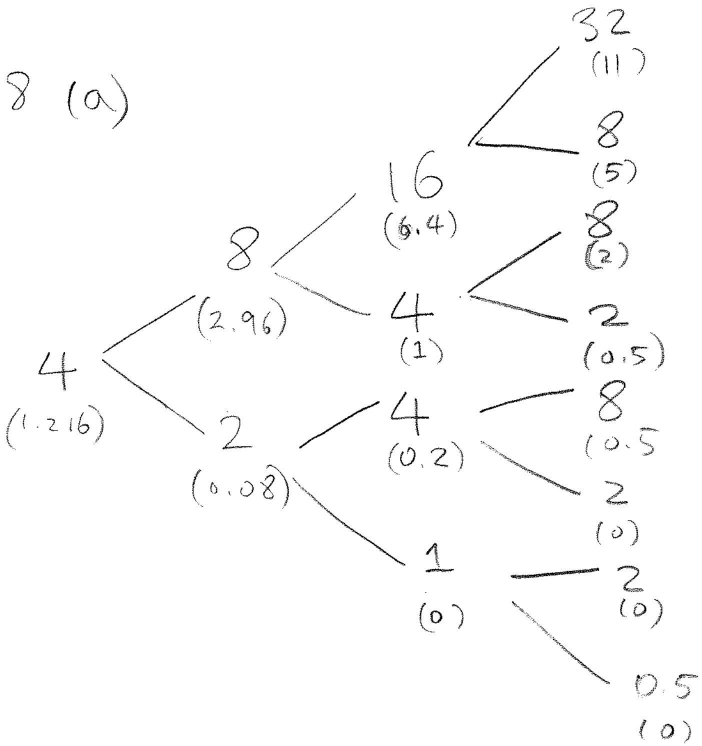
| | Cash | MS | NS | Option |
|---|---|---|---|---|
| win | 1 | 20 | -20 | 0 |
| draw | 1 | 10 | 5 | 5 |
| lose | 1 | -16 | 10 | 10 |

If we can replicate the option

$$\begin{cases} x + 20y - 20z = 0 \\ x + 10y + 5z = 5 \\ x - 16y + 10z = 10 \end{cases} \Rightarrow \begin{cases} x = 6 \\ y = -\frac{1}{6} \\ z = \frac{2}{15} \end{cases}$$

Therefore, it's verified that the option price should be 6.

6.8 (a)




Notice that $V_3 = (A_3 - 4)_+$

$V_2(HH) = 0.4\,(V_3(HHH) + V_3(HHT)) = 0.4(11+5) = 6.4$

$V_2(HT) = 0.4\,(V_3(HTH) + V_3(HTT)) = 0.4(2+0.5) = 1$

Likewise, we can fill in $V_2(TH) = 0.2$, $V_2(TT) = 0$

$V_1(H) = 0.4\,(V_2(HH) + V_2(HT)) = 2.96$

$V_1(T) = 0.4(0.2 + 0) = 0.08$

$V_0 = 0.4\,(2.96 + 0.08) = 1.216$

$\Delta_2(HH) = \dfrac{11-5}{32-8} = \dfrac{1}{4}$

$\Delta_2(HT) = \dfrac{2-0.5}{8-2} = \dfrac{1}{4}$

$\Delta_2(TH) = \dfrac{0.5-0}{8-2} = \dfrac{1}{12}$

$\Delta_2(TT) = 0$

$$\Delta_1(H) = \frac{6.4-1}{16-4} = \frac{9}{20}$$

$$\Delta_1(T) = \frac{0.2-0}{4-1} = \frac{1}{15}$$

$$\Delta_0 = \frac{2.96-0.08}{8-2} = \frac{12}{25}$$

(b) $V_0 = E^*\left[\frac{V_3}{(1+r)^3}\right] = \left(\frac{4}{5}\right)^3 \cdot \frac{1}{8}(11+5+2+0.5+0.5+0+0+0)$

$= 1.216$

$\square$.

Additional Problem :

(a) $V_p - V_c = \frac{K}{1+r} - S_0$

$\Rightarrow V_p = \frac{K}{1+r} - S_0 + V_c \geqslant 0$

$\Leftrightarrow V_c \geqslant S_0 - \frac{K}{1+r}$

$> S_0 - K$ since $\frac{1}{1+r} < 1$

$\square$

(b) If we exercise at time 0, we get $\max\{K-S_0, 0\}$

If we exercise at time 1, we get $V_p$

- Suppose $r=0$

when $K \leqslant S_0$, $\max\{K-S_0, 0\} = 0$ , $V_p = V_c + K - S_0 \geqslant 0$ } Optimal at time 1.

when $K \geqslant S_0$, $V_p = V_c + K - S_0 \geq K - S_0 = \max\{K-S_0, 0\}$

So it's optimal to hold until time 1 as well.

Now consider $r > 0$. Then it's not guaranteed that $V_p \geq K - S_0$

(we only have $V_p \geq \frac{K}{1+r} - S_0$). Thus the argument breaks down.

However, in part (a) $\frac{1}{1+r} \leq 1$ is always true so the result

in part (a) still holds regardless of the value of $r$.