Math 4740: Homework 10

Due Friday, May 6 in class.

Textbook exercises (from section 6.8): 6.5, 6.8. In exercise 6.8, to compute the replicating portfolio, use formula (6.13) from section 6.2. If you are not sure how to solve this problem, look at examples 6.2 and 6.3 from section 6.3.

Additional problem:

1. Recall the put-call parity formula (Theorem 6.6): The values V_P, V_C of European put and call options with the same strike price K and expiration time N are related by

$$V_P - V_C = \frac{K}{(1+r)^N} - S_0,$$

where r is the interest rate and S_0 is the stock price at time 0. In this problem we will assume that $N = 1$.

(a) Suppose you hold an *American* call option with strike K. Its value is $\max\{S_0 - K, 0\}$ if you exercise now or V_C if you wait until time 1, so the overall value is whichever of those two payoffs is greater. Clearly if $S_0 < K$ then it is optimal to hold the option until time 1, since $V_C \geq 0$. If $S_0 \geq K$, use the put-call parity formula along with the fact that $V_P \geq 0$ to demonstrate that $V_C \geq S_0 - K$, which implies that it is still optimal to hold the option until time 1.

(b) When the interest rate is $r = 0$, use a similar argument as in part (a) to prove that it is also optimal to hold an American put option with strike K until time 1. When $r > 0$, explain why this argument breaks down while the reasoning you used in (a) is still valid.