Math 4740: Homework 3

Due Friday, February 19 in class.

Textbook exercise (from section 1.12): 1.13.

Additional problems:

1. Here is the transition matrix for a Markov chain on the state space $\{1, 2, 3, 4, 5\}$:

$$P = \begin{bmatrix} 0.2 & 0.8 & 0 & 0 & 0 \\ 0.8 & 0.2 & 0 & 0 & 0 \\ 0.3 & 0.1 & 0 & 0 & 0.6 \\ 0 & 0 & 0 & 0.4 & 0.6 \\ 0 & 0 & 0 & 0.3 & 0.7 \end{bmatrix}$$

(a) List the communicating classes of recurrent states. For each communicating class C_i, let π_i be the stationary distribution for P that is supported on C_i (meaning that $\pi_i(x) = 0$ for all $x \notin C_i$). Use formula (1.8) from section 1.4 of the textbook to determine each π_i.

(b) Characterize all the stationary distributions of the Markov chain.

(c) Let $\mu = \begin{bmatrix} a & b & c & d & e \end{bmatrix}$ be a probability distribution on the state space. Provide a formula for $\lim_{n\to\infty} \mu P^n$ in terms of a, b, c, d, e. Use the behavior of the Markov chain to explain why this formula is correct.

(d) Suppose the third row of P is replaced with $\begin{bmatrix} 0.3 & 0 & 0.1 & 0 & 0.6 \end{bmatrix}$. Repeat part (c), again explaining the formula using your understanding of the Markov chain.

2. Let P be the transition matrix for a Markov chain. Suppose $\lim_{n\to\infty} P^n$ exists, and denote this limiting matrix by P^∞. Prove that every row of P^∞ is a stationary distribution for the Markov chain. *Hint:* What is $P^\infty P$?