3-MANIFOLDS

JASON MANNING
SCRIBED BY IAN PENDLETON
MAY 9, 2016

Definition. (M, R_+, R_-, γ) is a **sutured manifold** if γ is a union of annuli and tori in ∂M. $\partial M \setminus \mathring{\gamma} = R_+ \sqcup R_-$, and each annulus component of γ meets both R_+ and R_-.

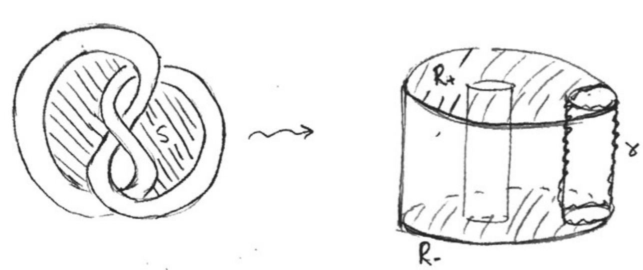
Foliations POV: Sutures are the part of ∂M to think of as transverse to $\mathcal{F}$ for some (imaginary) foliation $\mathcal{F}$ compatible with the sutured manifold structure. $R_\pm$ should be thought of as leaves.

Example. If $M \to S^1$ is a fiber bundle, the foliation by fibers is ∂M so we should start with $\partial M = \gamma$.

Definition. A **decomposing surface** for (M, γ) is a surface $(S, \partial S) \subset (M, \partial M)$ so that for each component σ of γ, $\partial S \cap \sigma$ is either

(i) a disjoint union of transverse arcs connecting R_+ to R_-
(ii) a disjoint union of parallel, essential (in γ) loops.

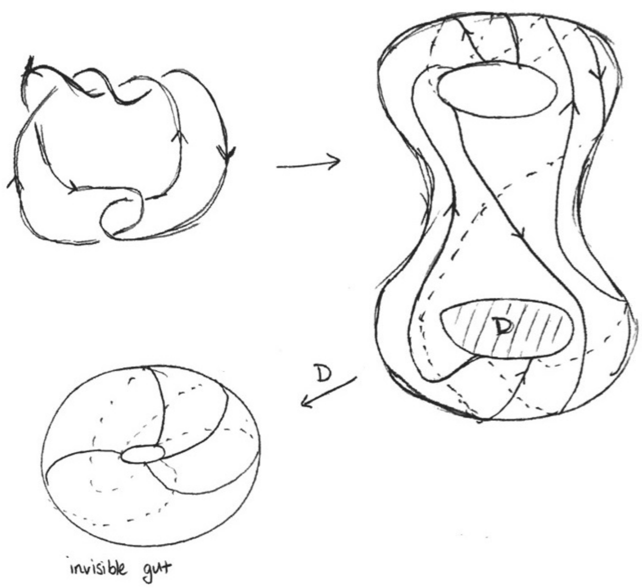
Definition. A sutured manifold (M, R_+, R_-, γ) is **taut** if M is irreducible, R_+ and R_- are norm-minimizing in $H_2(M, \gamma)$, R_+ and R_- admit no compression disks. If $(M, \gamma), (M', \gamma')$ are both taut, then $(M, \gamma) \rightsquigarrow (M', \gamma')$ is a **taut sutured manifold decomposition**.



In the figure above we take γ to be the torus boundary of a neighborhood of the trefoil knot K for $M = \overline{S^3 \setminus N(K)}$. Then the sutured manifold decomposition along the Seifert surface for K gives a fibered taut sutured manifold decomposition.

Example. Now we consider an example of a non-fibered knot complement, the stevedore knot 6_1.

Date: May 9th, 2016.



In this case the Seifert surface S is norm-minimizing. The knot complement is fibered if and only if cutting (M, γ) along S gives a product sutured manifold.

Note: $S^3 \backslash \mathring{N}(S)$ is a genus-2 handlebody.

Definition. The **index** of a decomposition disk D transverse to γ in (M, γ) is $\max\{0, \#(\partial D \cap \gamma) - 2\}$.

Definition. A disk D in (M, γ) is **admissible** if $\#(\partial D \cap \gamma) = 2$. An annulus is **admissible** if it has one ∂-component in R_+ and one in R_-. An **admissible decomposition surface** is a disjoint union of admissible disks and annuli.

Lemma. *Suppose* $(M, \gamma) \overset{S}{\rightsquigarrow} (M', \gamma')$ *is a taut sutured manifold decomposition with* S *admissible. Then* (M, γ) *is a product sutured manifold if and only if* (M', γ') *is a product sutured manifold.*

In our 6_1 knot complement example, the π_1 image of $R_+ \neq \pi_1(M_2)$. Therefore the 6_1 complement is not fibered. (We could also have seen this through the Alexander polynomial).

Definition. Let (M, γ) be a sutured manifold and $S \subset (M, \gamma)$ admissible. Let (M', γ') be obtained by sutured manifold decomposition along S. A **window** of (M, γ) is a product component of (M', γ'). A **gut** is a non-product component.

A gut is **invisible** if $\pi_1(\text{gut}) \to \pi_1(M) \to H_1(M; \mathbb{Q})$ is the zero map.
Working towards the fibering theorem:

Definition. Let G be a group. G is RFRS if there is a nested sequence of $G_i \lhd G$.

$$G = G_0 > G_1 > \cdots$$

so that:
(1) $\bigcap G_i = \{1\}$
(2) $[G : G_i] < \infty$
(3) $G_i \to G_i / G_{i+1}$ factors through $H_1(G_i)$/torsion.