MATH 2930 - Extra Credit 4

Monday, February 22, 2010
10:31 AM

3. Courtesy of Claire Paduano

3. (a) β= birth rate, δ= death rate

$$\delta \propto \beta \propto \frac{1}{\sqrt{P}}$$

so if

$$P'(t) = (\beta - \delta)P$$

$$P'(t) = (k_1 \frac{1}{\sqrt{P}} - k_2 \frac{1}{\sqrt{P}})P$$

$$P'(t) = (k_1 - k_2)\sqrt{P} = k\sqrt{P}$$

$$\int \frac{dP}{\sqrt{P}} = \int k dt$$

$$2\sqrt{P} = kt + C_0$$

$$\sqrt{P} = \frac{1}{2}kt + C$$

@t=0, P = P_0

$$\sqrt{P_0} = C$$

$$\sqrt{P} = \frac{1}{2}kt + \sqrt{P_0}$$

$$P(t) = (\frac{1}{2}kt + \sqrt{P_0})^2$$

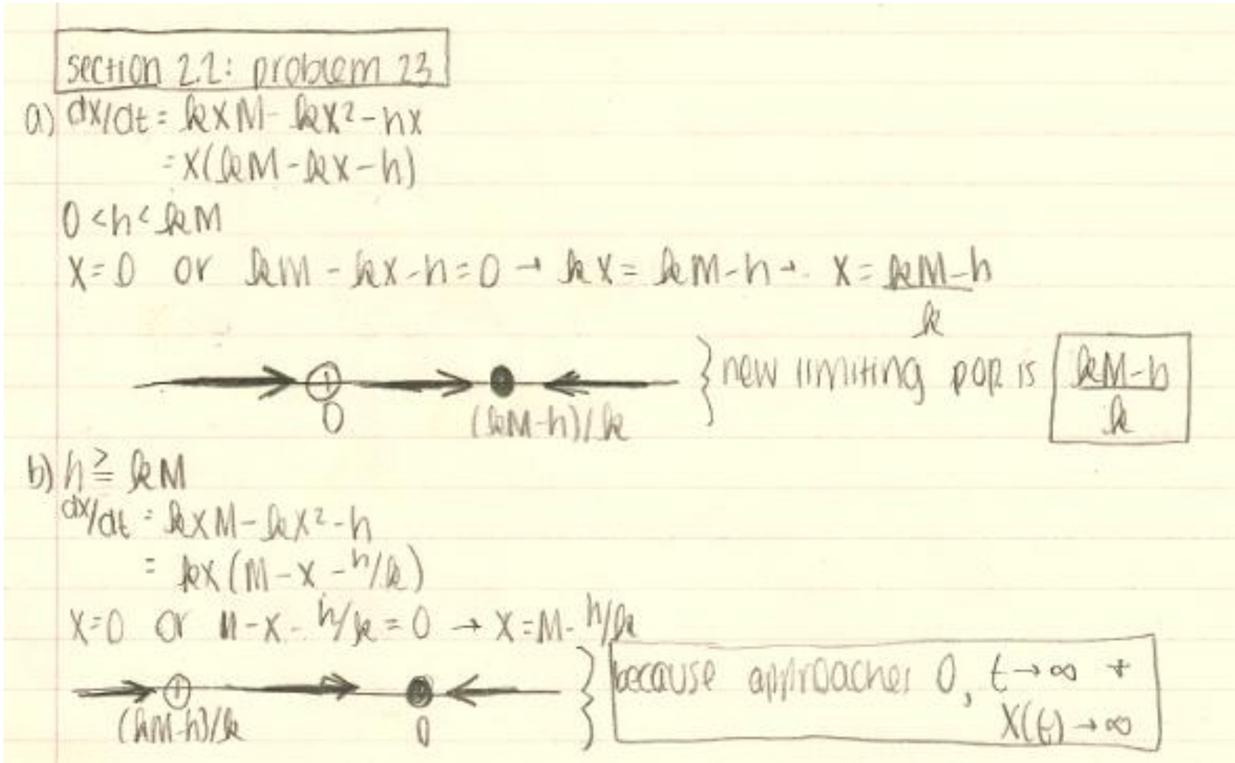
(b) after 6 months, there are 169 fish

$$169 = (\frac{1}{2}k(6months) + \sqrt{100})^2$$

Solving for k, you get k= 1 fish/month → $P = (\frac{t}{2} + 10)^2$

So at t= 1 year= 12 months, P=256 fish

4. Courtesy of Pam Snodgrass & George Lu



Section 2.2: problem 23

a) $dx/dt = kxM - kx^2 - hx$

$= x(kM - kx - h)$

$0 < h < kM$

$x = 0$ or $kM - kx - h = 0 \rightarrow kx = kM - h \rightarrow x = \frac{kM-h}{k}$

} new limiting pop is $\boxed{\frac{kM-h}{k}}$

b) $h \geq kM$

$dx/dt = kxM - kx^2 - h$

$= kx(M - x - h/k)$

$x = 0$ or $M - x - h/k = 0 \rightarrow x = M - h/k$

} because approaches 0, $t \rightarrow \infty$ + $X(t) \rightarrow \infty$

5. Courtesy of Lee Carlaw

a).

By Newton's second law we have:

$$\frac{d^2r}{dt^2} = \frac{-GM}{r^2}$$

Since the independent variable, t, is missing from the RHS of the equation, we can use the substitution method described in 1.6:

$$v = r' = \frac{dr}{dt} \Rightarrow \frac{d^2r}{dt^2} = r'' = \frac{dv}{dt} = \frac{dv}{dr} \cdot \frac{dr}{dt} = v\frac{dv}{dr}$$

$$v\frac{dv}{dr} = \frac{-GM}{r^2}$$

Integrating both sides:

$$\frac{1}{2}v^2 = \frac{GM}{r} + C$$

Since we know that $r'(0) = v_0$ and $r(0) = R$ we solve C to be:

$$C = \frac{1}{2}v_0^2 - \frac{GM}{r} \Rightarrow v^2 = v_0^2 + 2GM(\frac{1}{r} - \frac{1}{R})$$

Solving for r:

$$\frac{2GM}{R} - v_0^2 = \frac{2GM}{r}$$

$$\frac{R}{2GM - v_0^2 R} = \frac{r}{2GM}$$

$$\boxed{r_{max} = \frac{2GMR}{2GM - v_0^2 R}}$$

After plugging in $v_0^2 = \frac{2GM}{R}$ we immediately see that the projectile makes it to $r = +\infty$ and thus is able to continue out of earth's gravitational pull.

b).

Plugging in $100,000m$ into the v^2 equation where $r = r + R$ (since we initially start at the earth's surface) we have:

$$0 = v_0^2 + 2(6.67x10^{-11})(5.97x10^{24})(\frac{1}{7.378x10^6} - \frac{1}{6.378x10^6})$$

$$\boxed{v_0 = 1400m/s}$$

c).

Plugging in $v_0 = 0.9\sqrt{\frac{2GM}{R}} = 7111.3m/s$ into the derived r_{max} equation, we get:

$$\frac{2GMR}{2GM(1 - 0.81)} = r_{max} \Rightarrow \boxed{r_{max} = \frac{100R}{19}}$$