Chapter 2: Literature Review

The use of writing tasks in instruction has been increasingly discussed and researched since the late 1970s, with the beginning of the Writing Across the Curriculum movement. The use of writing in learning and teaching mathematics has been documented and studied starting in the mid to late 1980s. Most of the literature on writing to learn can be divided into three categories: theoretical rationale for the use of writing, research on the results of using writing in the classroom, and examples of writing tasks that teachers have used in their classrooms and the apparent outcomes. This literature explains why teachers use writing in instruction, demonstrates the state of research that motivates this study and provides further examples of writing tasks to supplement those collected for this study. Knowing the goals for the use of writing in instruction gives some inspiration for a classification since the theory of how writing affects learning may motivate the inclusion of certain variations so that these theories can be tested. Understanding the state of the research on writing in mathematics instruction shows us that greater description of tasks may motivate more specific research on the use of writing, clarify the results of the research and make the results easier to implement. Finally, examples of writing in the literature can be used to provide more examples of categories of variations found within the collection.

This chapter surveys the theoretical underpinnings for the use of writing in instruction, answering the question of why it makes sense to use writing to help students learn. Then the research regarding the student outcomes from various types of writing tasks is discussed. The state of this research justifies the need for a classification of writing tasks and the method by which it is accomplished. This literature also gives theoretical rationale and results which guide certain aspects of the classification. The third category of the literature, containing examples of writing

tasks, will be used as needed later to provide examples of the classification beyond the collection.

Theoretical rationale: why use writing?

First, I will consider the literature on writing in general, and then consider research about the use of writing in mathematics in particular. This moves from a general to more specific context.

General Theories About How Writing Affects Learning

Ties Between Thought and Language

The assumptions and research about the ties between thought and speech are important background to the use of writing in instruction. One of the most influential theorists in this area is Vygotsky, who worked in Russia during the 1920s and 1930s, although his work was not discovered in the West until the 1960s. Vygotsky refers to language as the "social means of thought" (1962, p. 51). His theory was based on a close interrelation between thought and speech; not that one preceded the other, but that development in one could lead to development in the other: "The relation of thought to word is not a thing but a process, a continual movement back and forth from thought to word and from word to thought" (1962, p. 125). This validates the importance of using language to help develop thoughts, and therefore concepts, in the learning process. Even more strongly, Vygotsky referred to language as the embodiment of thought: "Thought undergoes many changes as it turns into speech. It does not merely find expression in speech; it finds its reality and form" (1962, p. 126). Verbalization in some form, then, is necessary to substantiate and organize thought. Vygotsky also notes that speech and thought can connect different ideas: "Thought is not merely expressed in words; it comes into existence through them. Every thought tends to connect something with something else, to establish a relationship between

things" (1962, p. 125). Vygotsky's theory, therefore, establishes the importance of communication and verbalization of ideas in developing deep understanding of the concepts, because it places the development of thought – its existence and its structure – side by side with the communication of ideas.

Uniqueness of Writing as a Form of Communication

Writing is considered a special form of this communication. Vygotsky considered written language a different form of communication than that of verbal language. He thought of writing as a form of monologue, a form which "is a complex formation; the linguistic elaboration can be attended to leisurely and consciously" (1962, p. 144), unlike the dialogue of verbal speech, which is more spontaneous and, therefore, simple. "Planning has an important part in written speech, even when we do not actually write out a draft. Usually we say to ourselves what we are going to write; this is also a draft, though in thought only" (1962, p. 144). Vygotsky believes this planning and organization are inherent in writing, and make it more conscious or deliberate than other forms of speech. According to Vygotsky, writing also requires much more attention to the formal meanings of words and a greater number of words since the intended audience was not present, and needs fuller explication to garner the meaning intended by the writer. This is in contrast to inner speech, which is directed to the self and therefore needs no explanation or added context. This ability to detach oneself from the situation and explain it to another requires a higher level of abstraction. Likewise, the extra layers of symbols and structure – the words on the page and the structure of each sentence – require a higher level of abstraction in written speech than in verbal speech (Vygotsky, 1962). Vygotsky also recognizes the interplay between revision of the written document and the revision of thought: "The evolution from the draft to the final copy reflects our mental process" (1962, p. 144). Therefore, Vygotsky believed that writing required more planning and conscious

effort, as well as greater abstraction, than other forms of communication. The nature of written revision might also provide a concrete expression of changing thought.

Emig (1977) also states reasons why writing is unique amongst other forms of verbal communication. Writing is compared to the other languaging processes of reading, listening and talking. Emig notes that only in writing does one originate an idea and record it graphically. Similarly, when listening, the auditor neither originates nor records an idea. This is in contrast to reading, in which one creates meaning from an idea that is recorded graphically, but does not originate the idea (Emig, 1977). Talking may serve as pre-writing since ideas originate in this process, but Emig suggests it is inaccurate "to say that writing is talk recorded" (Emig, 1977, p. 123). Emig (1997) lists the following differences between writing and talking:

- writing is a learned behavior, and therefore artificial
- writing is usually slower than talking, allowing more opportunity for review, revision and a greater depth of cognition
- writing is a "technological device" (1977, p. 124), rather than natural
- writing "must provide its own context" rather than depend on the situation
- writing is usually to an absent party
- "writing tends to be a more responsible and committed act than talking"
- writing includes both "process and product" and is more easily a "form and source of learning"
- writing is afforded a permanence not normally given to talking.

In this way, Emig delineates the uniqueness of writing as a form of communication.

Zinsser's theories about writing to learn agree closely with Vygotsky and Emig. His understanding of the connection between thought and language reflects that of Vygotsky: "Writing and thinking and learning were the same process" (Zinsser,

1988, p. ix). He also shares Vygotsky's belief that writing is a deliberate activity and therefore "organizes and clarifies our thoughts" (p. 16). However, Zinsser focuses on writing as a means to see explore one's own thoughts, as well as a means of developing thought: "Writing is how we think our way into a subject and make it our own. Writing enables us to find out what we know – and what we don't know – about whatever we're trying to learn" (p. 16). Zinsser also extends this identification between writing and thinking to its implications for instruction in writing in the disciplines: "there's little point in having a teacher clean up the messy syntax in a chemistry paper if he can't also clean up the messy chemistry. The indivisibility of language is what gives writing its authority and its majesty" (p. 53). Therefore, writing in a subject area should be considered by those who understand the content, and not just writing experts.

Unique Correspondence Between Writing and Learning

Emig (1977) also theorized about the uniqueness of writing as a method for learning. She developed close parallels between writing processes and learning processes, and emphasized the uniqueness of writing as a use of language. Emig offers four ways that writing and learning processes correspond: they are "multirepresentational and integrative" (1977, p. 128), have similar types and uses of feedback, promote connections, and are "active, engaged and personal" (1977, p. 128). First of all, Emig notes that learning occurs through a variety of representations: it can be enactive – what we do – iconic – what we see – or symbolic – through the words we use. Writing includes all three representations as we use our hand to write, see the result on the paper, and think of the words we wish to write, so that the writing is reinforced through the multiple representations. Also, Emig notes that writing uses the full extent of the brain, thereby integrating these functions with each other and the necessary functions of hand and eye. Secondly, Emig notes that when writing we

have immediate feedback, based on what we have already written, as well as long-term feedback through the written record. We can respond to this feedback in a fashion similar to our use of short-term and long-term feedback in the learning process. This is closely tied with Vygotsky's theories about revision of writing, since the external nature of writing can begin a dialogue between the written product and thought which allows both to change. Thirdly, Emig draws on Bruner's axiom of learning, "We are connective" (in Emig, 1977, p. 125), and Vygotsky's arguments about the need for "deliberate structuring of the web of meaning" (Vygotsky, 1962, p. 100) to show how both learning and writing make connections. She notes that this is especially true in writing, which requires the writer to expound on these connections so that they are clear to the reader. Finally, just as learning must be "active, engaged, and personal" (Emig, 1977, p. 128), so must writing. She also notes that both proceed at their own pace, that is they are "self-rhythmed" (1977, p.128). We can speed or slow our writing or revision to be most effective for our learning: "writing can sponsor learning because it can match its pace" (Emig, 1977, p. 126).

Different Types of Writing and Thinking

Vygotsky and Emig's theories appear to apply most specifically to published writing which is shared with others and needs to convey its meaning precisely. A contrasting type of writing is described by Peter Elbow. He differentiates between "freewriting and fast expository writing" and "careful, conscious, critical revising" (1986, p. 55). He recognizes these two types of writing as contributing to different types of thinking: first-order thinking, which is "intuitive and creative," uncontrolled, leading to a sense of discovering one's own thoughts, and second-order thinking, which is "conscious, directed, controlled thinking" (p. 55). Elbow believes that first-order thinking is likely to contain more ideas, use livelier language, and may give hints of organization in ways that second-order thinking cannot, and therefore is useful

in first-drafts or pre-writing exercises. Second-order thinking, however, allows the writer to reflect upon the first order thoughts and evaluate their meaning and organization. Therefore, second-order thinking is most helpful when revising rather than writing in the first place. However, the writing that is associated with first-order thinking – freewriting and first-drafts – is tied more closely to speech than the finished product if we consider the differentiation Vygotsky and Emig make between verbal and written speech. This type of writing is parallel to speech in that it is more spontaneous and faster. Both types of writing, however, are tied by Elbow to thinking, following Vygotsky's ideas of how thought and language are related. However, this intimates that there may be a correspondence between different types of writing and different thinking processes that a teacher may want to exploit in instruction. If the writing is not to be shared with others, there is not as much need for deliberation in the words used, or the need for as many words to explain. If we focus only on first-order thinking, there is also no need for revision, so the feedback from the writing may not be as deep, limiting the interplay between thought and word.

Zinsser makes a similar distinction between two types of writing: "exploratory writing: writing that enables us to discover what we want to say" and "explanatory writing: writing that transmits existing information or ideas" (Zinsser, 1988, p. 55-56). The former can be closely identified with Elbow's first order writing, but the latter focuses more on the product than process of the work. The writing-process model in McIntosh & Draper (1997) highlights some of the differences between these types of writing that may have bearing on what is learned by students. They mention five stages of writing: prewriting, composing (or drafting), revising, editing (or proofreading), and publishing (or sharing) (1997, p. 2). Exploratory writing only makes use of the first two stages and therefore may promote different types of interplay between thought and language.

Summary of Assumptions about Writing and Thinking

Applebee (1987, p.577) summarizes the roles of writing in thinking that are found in the literature as

> "(a) the permanence of the written word, allowing the writer to rethink and revise over an extended period;
>
> (b) the explicitness required in writing, if meaning is to remain constant beyond the context in which it was originally written;
>
> (c) the resources provided by the conventional forms of discourse for organizing and thinking through new ideas of experiences and for explicating the relationships among them; and
>
> (d) the active nature of writing, providing a medium for exploring implications entailed within otherwise unexamined assumptions."

These four roles are tied closely to the uniqueness of writing as a form of communication and its parallels with the processes of learning. They become central reasons to use writing as an instrument of learning and, therefore, should be taken into account in any description of writing tasks. Since there is variation in the different types of writing as described by Elbow and Zinsser, some of these elements may be more or less present in a different type of writing and affect resulting student learning. Issues like permanence of the writing, whether it is revised, how precise the writing needs to be, how much organization is required, and whether students actively participate need to be considered when describing writing tasks. These issues affect the thinking processes that students use and are affected by the parameters of the task.

Affect of Experience on Writing Within a Discipline and Revision

The effectiveness of writing as a learning tool may be affected by the experience of the writer. The nature of writing in different disciplines is considered by Williams and Colomb (1990). Research shows that writing skill in one discipline may

not necessarily transfer to writing skill in other disciplines. That is, higher-level cognitive processes may depend not only on generic and abstract processing skills, but also on knowledge of and experience with the content area. In an experiment by Voss and others (1985), they found that senior chemistry professors behaved similarly to students taking a first course in Soviet affairs when considering the agricultural problems of the Soviet Union. Without knowledge of the discipline, the solutions tended to be more concrete and have shorter chains of argument than experts in the field did. That is, even though the chemistry professors had evidenced high-level thinking in their own field, it did not transfer to that of Soviet affairs. Williams and Colomb note that such results point out that a linear view of writing development is therefore detrimental, since it does not take into account the need for discipline specific knowledge. Instead they suggest that progress be considered as the joining of a community, where the writer is socialized into the discipline. Even an experienced writer needs to learn the ways of thinking particular to a new discipline, and what things should be tacit in the writing, rather than explained. From this research, we can see the motivation for the Writing in the Disciplines movement. According to Williams and Colomb, learning how to write in a discipline is a necessary part of joining the learning community.

Similarly, Sommers (1980) finds that experience with writing makes a difference in the revision strategies of the writer. These differences center on the revision strategies of the different groups, and highlight different expectations for structure, audience and meaning in the writers. Inexperienced writers make most of their changes on the word level, substituting for a word or deleting unnecessary words, while experienced writers make a majority of their changes on the sentence level, reordering and adding words and sentences as well as substituting and deleting. Sommers draws the conclusion that inexperienced writers are focused more on words

than meanings in their writing. Experienced writers tended to use their first drafts to discover what they wanted to say. They then reorganize this information, and shape it with a reader in mind. Inexperienced writers, however, are more likely to feel that they are trying to write out a meaning that is already predetermined, merely translating thoughts in their head to paper, and do not see that they "modify and develop perspectives and ideas" (p. 382) as they revise. This has implications for using writing as a means for learning. Many of the benefits that are theorized to be associated with writing depend on the interplay between thought and word, with the revisions of one leading to the revision of the other. If students are unclear that revision of meaning is possible, these benefits may be very difficult to achieve. The benefits of Elbow's first-order writing may still hold; even here, however, if students think that they are writing out meanings that are already well-formed in their head, they may lose the sense of discovery that is important with this type of writing.

Mathematics Education Motivations for Writing

The movement to use writing in mathematics reflects both the Writing to Learn and the Writing in the Disciplines strands of the Writing Across the Curriculum movement. As students write, "they communicate to learn mathematics, and they learn to communicate mathematically" (NCTM, 2000, p.59). The NCTM Standards promote Communication as one of its five process strands, making it central in its view of mathematics and mathematics instruction, and this view is very influential in the mathematics education community. Both the use of writing as an instructional tool and the importance of mathematical communication are recognized, and reflect the two strands of the Writing Across the Curriculum movement. A number of factors have contributed to the growth of writing in mathematics instruction: a desire to promote deeper understanding of mathematics, the influence of educational

constructivism, cognitive studies research, changes in both the perception of the discipline and its emphasis on writing and various pedagogical considerations.

Promoting Understanding and Thinking

This emphasis on writing and communication followed a period of greater emphasis on meaning and conceptual understanding within the discipline. Richard Skemp was a leader in this emphasis on meaning, and differentiated between two types of understanding: relational understanding and instrumental understanding. He defined relational understanding as "knowing both what to do and why" (1978, p. 9), in contrast with instrumental understanding, which he defines as "the possession of …a rule, and ability to use it." (1978, p. 9) Promoting relational, or conceptual, understanding continues to be an issue of concern in the mathematics education community, and the use of writing is seen as one way to meet this need for deeper understanding. The use of writing to work toward a deeper understanding of the subject parallels the work of Zinsser (1988) and the Writing to Learn movement..

As is reflected in the general rationale for Writing to Learn, it is recognized in mathematics that "through communication, ideas become objects of reflection, refinement, discussion, and amendment" (NCTM, 2000, p. 59). This emphasizes the external nature of a written task after it is completed, allowing students to act on it as something external, even though it represents their internal thoughts. This leads to metacognition, which is recognized as an important part of the learning process. Metacognition is the process of thinking about one's own thinking: knowing what one is thinking and self-regulating it. According to the Vygotskian dialectic between writing and thinking, writing will give insight into thinking. Once this representation of thinking is on paper, students can reflect on their thinking as if it were something outside of themselves, therefore allowing metacognition. Pugalee writes that "[w]riting helps build thinking skills for mathematics students as they become

accustomed to reflecting and synthesizing as parts of a normal sequence involved in communicating about mathematics" (1997, p. 308)

Educational Constructivism

The development of the educational theory of constructivism in mathematics also has implications for why people choose to use writing in mathematics instruction. This theory purports that students construct their own mental representations of concepts based on the input they are given and that these representations are personal, so they will not be mirror images of what is presented. According to Davis (1993), this implies that "the task of teaching is to help students build up larger and more powerful collections of basic representational metaphors, and to help students develop more powerful ways of thinking."(p. 295). This focuses on providing opportunities for students to experience and discover the meaning of the mathematics, in contrast to the teaching style Davis terms "telling" and "drill and practice" (p. 296). NCTM recognizes that writing, along with other forms of communication, "helps build meaning and permanence for ideas" (NCTM, p. 59). A constructivist viewpoint is also inherent in Whitin & Whitin's approach to communication in the classroom. They posit their use of writing and talking as a way of "respecting children as sense-makers" (2000, p. 1), encouraging them to learn how to think and construct their own meanings. Elbow's freewriting and other writing activities that promote first-order thinking would provide a chance for discovery. Students would also be able to explore their own meanings and thoughts in writing; Vygotsky's emphasis on the connection between thought and language is a vital link here. Written work is also something that can be shared with other students and teachers so that they can be exposed to a number of ways of thinking about the concepts, thereby promoting a greater wealth of metaphors for the concept.

Cognitive Studies Research

Research in cognitive studies that explains how our brains process ideas also motivate the use of writing in mathematics. Emig is not the only researcher to link writing to different kinds of brain activity. Krpan (2000) justifies the use of writing based on brain research that suggests that writing can help retention of material and connections between it. She suggests that the personal nature of writing and the meaning students build in the process make the ideas more "emotionally charged," (p. 6) thereby making them more memorable. Likewise, the connections students make between their experiences and mathematical content would make more neural connections in the brain, allowing them to use what they know more easily. Also, the more time the brain has to process the information, the more it will solidify the connections it has made. Writing can slow students down to give them this time. The impact on the brain relies on the assumptions that writing helps students construct personal meaning, make connections between their experience and learning, and reflect on what they have learned.

Changes in Perceptions of the Discipline

A shift in the perception of mathematics as a discipline has also contributed to the desire to use writing in mathematics instruction. Many mathematics educators are trying to counter the stereotypical view of learning mathematics where students work alone, students passively accept the information teachers pass on and practice it with twenty similar problems, and the teacher is the authority on whether an answer is right or wrong. Based on these patterns, students form the belief that mathematics is soliatary, done by respective algorithms passed on by others, and requires no reasoning on their part. Writing is seen as one way to address these student misconceptions and change attitudes toward mathematics. Miller (1991) writes that mathematics instruction has been moving "away from formal correctness and finished

product and toward process, context and understanding" (p. 516). Writing, in its own process, can promote this different view of mathematics. Also, writing allows each student to make connections to his or her own experiences, giving more context. Therefore, mathematics becomes more personal and less teacher-centered. Students also gain ownership and confidence in their own thought processes rather than relying on the teacher to determine if an answer is right or wrong.

Growth in Disciplinary Emphasis on Writing

This shift in perceptions of mathematics coincides with a greater emphasis on communicating mathematics and professional writing in mathematics. Proponents of writing in mathematics hope that it will not only help students learn concepts and change their attitudes, but that it will help them learn to communicate mathematically. Some of the qualities particularly important in mathematical communication include the ability to make an argument or proof, and the need to be precise. Through communication, it is expected that students "learn to be clear and convincing" (NCTM, 2000, p. 59) when they need to explain their reasoning to others. Writing may encourage greater clarity and precision than other forms of communication; Vygotsky recognized that writing must include its own context to communicate, while talking occurs in context and needs less explication.

When Zinsser wrote *Writing to Learn* in 1988, and a year before the first NCTM *Principles and Standards* document, emphasis on writing in instruction was just beginning. Zinsser's chapter on the use of writing in mathematics focuses on only one teacher's efforts. This teacher, Joan Countryman, later wrote her own book on writing to learn mathematics (1992). The use of writing in mathematics followed the beginning of the Writing Across the Curriculum movement by a decade. This is not surprising, since mathematics is frequently stereotyped to have a distant relationship with spoken and written language because of its use of symbols. Zinsser himself was

surprised that writing could be used in mathematics, and had to turn to a secondary school teacher to find examples, although the rest of the book focused on college examples. However, through his time with Countryman, he recognized that mathematics has its own disciplinary language, but one that is perhaps more difficult to learn than that of other disciplines. It is also interesting to note that the chapter on using writing in mathematics is the only chapter that uses exclusively student examples of writing rather than professional examples, while only one other chapter, that on chemistry and physics, uses any student writings tasks. One can infer that perhaps Zinsser was less comfortable with the professional literature in mathematics or that there was less attention paid to writing by the professional mathematics community itself.

This latter assertion can be supported by surveying the professional literature on writing in mathematics. Only one of the professional mathematics societies had a guide for writing in mathematics before 1985: The American Mathematics Society published the guide, *A Manual for Authors of Mathematical Papers,* in 1962. Starting with Gillman's guide published by the Mathematical Association of America in 1987, however, there have been a number of books written about writing mathematics well (Knuth, Larrabee, and Roberts, 1989; Higham, 1993; Krantz, 1997, etc.). It can be inferred that within the American mathematical community, the issue of writing mathematics well is taking more precedence now than it did thirty years ago. This is also reflected in the greater number of expository mathematics books that are being published. Therefore we can see that the use of writing in mathematics instruction coincided closely with a greater disciplinary interest in writing.

Pedagogical Reasons to Use Writing

Many of the reasons in the literature for using writing in mathematics are more pedagogical, rather than theoretical. Miller (1991) suggests that writing allows all

students to ask questions, even if they are too shy to ask in class. Also, students who have math anxiety and are better at other subjects may feel more comfortable writing about mathematics. This also connects to the research on multiple intelligences (Gardner, 1985). Writing provides a different means for students to approach their learning, through the verbal-linguistic intelligence, rather than the typical logical-mathematical intelligence associated with mathematics. Geeslin (1977), in comparing writing to talking, focuses on practical reasons rather than the correspondence between writing and learning that Emig considered. He mentions that writing, when compared to discussion, involves all students and leaves a permanent record that can be examined by the teacher or teacher and student together. Whitin & Whitin echo these differences: writing "[creates] a record of our thinking that we can analyze and reflect upon" (2000, p. 2) and helps students "develop a personal voice" (2000, p. 2). Burns (1995) lists three outcomes that the use of writing in mathematics might support: it "helps teachers evaluate how well the instructional program is supporting learning goals" (1995, p. 29), promotes "learning about individuals' understanding and skills" (1995, p. 30) and is a "vehicle for communicating with parents" (1995, p. 30). Most of these goals are teacher-oriented rather than learner-oriented, recognizing that writing allows the teacher to keep all students involved, assess students' work, both as a whole and individually, and incorporate parents in the learning process.

The idea that teachers can assess students through their written work deserves further examination. Beyond the fact that there is usually information from every student, there is an assumption that assessment through written or spoken language gives deeper and more detailed insights into a student's understanding than does traditional problem sets. The influence of constructivism can be seen here, in the assumption that a student can do correct exercises, but have incorrect thinking. Therefore, there is a need to find ways to assess students' thinking. "Writing and

talking are ways that learners can make their mathematical thinking visible." (Whitin & Whitin, 2000, p. 2) It is assumed that "if students can write clearly about mathematics concepts, then they probably understand them" (Miller, 1991, p. 516). Therefore, writing has become a significant means of assessing students' thinking and understanding.

The increasing prevalence and importance of standardized tests also provides practical motivation for using writing. Krpan (2001) believes that the deeper understanding of mathematics that grows out of writing about it helps students think about whether answers on standardized tests are plausible, as well as allowing them to identify more helpful strategies because they have thought about their own mathematical thinking. Also, the positive attitudes toward mathematics that writing may engender would reduce test anxiety. Another aspect of this is the increasing use of short-answer or free-response question formats on standardized tests that require that students write out their answers. For this reason, many teachers are using more writing in instruction to give students more practice, and books have been written to help teachers prepare their students for the writing required on these tests (e.g. Thomas, 2004).

Effects of Writing on Learning

With this wealth of theory and justification for why writing promotes learning, the dearth of research that supports these assumptions is surprising. There is a large literature of examples of writing tasks in mathematics, but few clear results that affirm the major assumptions about the outcomes of writing: writing helps students understand concepts better, writing promotes metacognition because it allows students to act on a written record of their own thinking, writing changes student attitudes toward mathematics. This lack of results is especially glaring if one looks at research

on writing in mathematics. Vygotsky performed a few experiments that appeared consistent with his theories, but this program of research was interrupted by his death. We will survey what is known about the effects of writing in instruction, both in the general and discipline-specific literature.

Recursive Process of Writing

One of the central results from research on writing has to do with the writing process. Although taught in a linear set of stages, Emig (1971) found that the writing process was recursive and complex. Applebee summarizes work in the same vein over the next decade as "emphasizing the essentially heuristic, problem-solving nature of writing about new material" (1987, p. 582). The recursive nature of writing would be consistent with Vygotsky's and Emig's claims that writing and thinking are interrelated. The close tie to problem-solving also suggests that there may be a close tie with mathematical ways of thinking. However, none of these process studies "provides useful evidence of how – or even whether – any such learning actually takes place" (Applebee, 1987, p. 583).

Different Tasks Result in Different Learning

One of the things that is clear in the literature is that the type of task has a significant effect on the types of learning which result (Applebee, 1987). Applebee (1987) reports that many studies show that the writing processes used differ based on the task, as well as on the individual writer. This difference in writing processes would be consistent with different learning according to the theories of Vygotsky and Emig, so these results are consistent with the theory. Applebee concludes, from a synthesis of research on writing as a way of reasoning, that the extent to which material is manipulated and the breadth of an activity result in different depth and content of learning. More manipulation, or deeper processing of the material, tended to increase recall and the ability to use the information in different contexts; this is

most evident if the material is tested in a manner similar to the manipulation. That is, questions following a reading passage have a positive effect on later questions, more so if the same questions are repeated. Similarly, short answer responses showed a greater increase in learning than multiple choice responses and reorganization of material also seemed to benefit later recall and use more than verbatim repetition. Applebee also noted that the breadth of content that the writing activities focused on made a difference in student learning; "writing tasks that focus on a limited problem are unlikely to lead to general effects" (1987, p. 586). Therefore, since writing tasks vary in the depth of processing they require and breadth of content, it is likely that different writing tasks will result in different learning.

Applebee refers to some classroom studies with control classes and treatment groups that varied only in their use of writing. The results show very little difference between the two groups, although the differences tended to favor the experimental group. Applebee notes that these studies are "more interesting for [their] application of writing activities to a variety of subject areas than for the strength of [their] findings" (1987, p. 588).

The same can be said for most of the studies of writing in mathematics, even twenty years later. Most of the existing literature tends to be descriptive of approaches to writing in mathematics and surveys of apparent results rather than studies of changes in students' learning as a result of writing experience (e.g. Beidleman, et al., 1995; Gopen and Smith, 1990; Price, 1989; etc.) Many of the articles are descriptions of one or two people's uses of writing in mathematics. These articles present ideas about why writing should be used and share the teachers' experience and instructional ideas.

Need for Mathematics-Focused Research

Mathematics educators could choose to base the implementation of writing in instruction solely on the general writing to learn research. However, I believe that writing in mathematics varies enough from writing in other subject areas that the general writing to learn research may not always apply. As the Writing in the Disciplines research implies, writing in every subject area has knowledge and conventions that must be mastered. The study of mathematics expects unique types of learning, and lends itself to different uses of writing than many other subjects. Precision is more highly valued than in most subject areas, and there is a specialized vocabulary. There are also subject-specific standards for justification and logic. With different learning goals, different writing activities may be necessary to target these goals, but neither the mathematics-specific writing activities nor the learning objectives are reflected in the general literature. Therefore, the literature cannot inform mathematics teachers which writing activities would be most helpful to promote the student learning they desire. Also, when this literature is applicable, it is not always accessible to mathematics teachers since they are not trained to teach writing. It is necessary to strengthen the ties between the writing that occurs in mathematics classrooms and the research on the effects of writing and to present this research in forums and ways that are accessible to mathematics teachers so that the writing will be useful for mathematics teachers. Therefore, I believe that research on writing as it occurs in mathematics classrooms is warranted.

Research on the use of writing in mathematics has only begun in the last twenty years. A decade ago, Clarke et al. (1993) stated that the writing movement in mathematics appeals to Vygotsky and Emig in their studies of writing and its effect on learning since there are no equivalent studies in mathematics. Shepard (1993), in recognizing the dearth of research in this area, proposed a model for the use of writing

in conceptual development, associating a progression of conceptual understanding with writing tasks that could help students advance in this progression. However, this work focuses solely on conceptual development, is not based on actual writing assignments, and does not take many of the factors already identified in the writing to learn literature into account. Most other studies are more practical and do not address issues of theory or actual student learning, focusing instead on particular implementations of writing and informal assessments of the benefits of writing.

Affects of Writing on Mathematics Learning

Beidleman, Jones & Wells (1995) describe their use of various writing assignments in Calculus and the results they saw. Students were given procedural quizzes twice a week, in-class writing assignments at least once a week, weekly out-of-class writing assignments that required creative thought and at least one writing question on each exam. They noted that students improved the quality of their writing over the course of the semester and a few students made significant improvements in their grades that they attributed in part to the writing assignments. However, no specific improvements in students' understanding were measured, and the multitude of writing activities would make it difficult to determine which tasks had which effects even if the effects had been measured. This was one of many articles written by teachers about how they used writing, the results they saw, and the changes they would make (see also Williams & Wynne, 2000; Sjoberg, Slavit and Coon, 2004).

Gopen and Smith (1990) write about their approach to teaching writing in mathematics. The focus is on making student writing clear, focused and understandable to a reader, and ascribes more to the Writing in the Disciplines than the Writing to Learn movement. Gopen and Smith identify a number of discipline specific errors in student writing and show how they used questions to help the writer think about how the reader will read the explanation to help student revise and

improve their writing. The approach is well-explained and the examples are clear, but there is no sense given of the scope of the improvement, and little attention is paid to the actual assignments.

The study of writing in mathematics has frequently been studied as teachers undertake action research in their own classroom. One such example is Stofferan (2005). She implemented a variety of writing tasks in one of her classes and compared pre- and post-treatment results on both a self-rating of mathematics ability and a survey of students' attitudes and approaches to mathematics problems. She also collected student evaluations of how writing affected their learning. She found that most students believed that writing in mathematics was beneficial to them and that it helped them understand mathematical concepts better. Also, many beliefs about mathematics showed changes, with students expressing more belief that mathematics was applicable, that there were multiple ways to solve a problem and that they liked mathematics. These effects were seen more strongly in the male subgroup, who also were also more likely to believe that their mathematical ability had improved during the course of the study. Although one of her goals in using writing was to increase problem solving skills, notably higher-order skills that require metacognition, no information was collected regarding the growth of students' abilities in this area because the strategies used to solve open-ended problems were difficult to measure.

Borasi and Rose (1989) studied a class of students who used journals in order to delineate potential benefits of journal writing. They used journals in a college algebra class, one entry per class, so that students could "reflect on and express feelings about mathematical content and the course, to provide input to the teacher, and to engage in a dialogue with the teacher" (1989, p. 350). Students were also asked to occasionally write in the journals in class. At the end of the semester, students evaluated their journal writing: how it affected their learning, their feelings about it,

the perceived benefits of journal writing and suggestions for improvement. Borasi and Rose identified three categories of benefits: benefits for the student, benefits to the teacher, and benefits for the student-teacher dialogue. They saw that journal writing could help students by a therapeutic sharing of emotions, increasing their content knowledge, improving their problem solving skills, and changing their beliefs about mathematics. Teachers could benefit through better assessment of and intervention in students' learning, and immediate and long-term improvements in the course structure and teaching approach. Dialogue between students and teachers could allow more individualized teaching, and a more relaxed classroom atmosphere as student-teacher relationship improved. They also correlated these benefits with the different types of writing that students did in their journals, although it was not clear whether these correlations were based on analysis of the relationship between students' journals and their evaluations or whether it was a theoretical analysis. Many of these benefits are closely related to those expected by the theories of Vygotsky and Emig regarding how writing interacts with thought and allows students to review their thoughts. It is unclear throughout the literature whether teachers' expectations for using writing are based on theoretical research or on students' evaluations of writing in mathematics. It is possible the effect is circular, since usually, as in the case of Borasi and Rose, students are given some rationale about why they are asked to do the writing assignment. Borasi & Rose justify their focus on students' perceived benefits rather than actual changes in learning because the find it "unrealistic" and "inappropriate" (p. 349) to seek empirical research on which learning is predicted by the assignment because the learning depends on the instructional context. Their approach is intended to clarify the opportunities to learn that journal writing provides. The work of Stofferan and Borasi & Rose therefore goes beyond that of Beidleman et al and Gopen

& Smith by formally assessing benefits of writing, but only the perceived benefits of the students.

Clarke et al. (1993), in a departure from this pedagogical and descriptive norm, study a group of 500 secondary students using a specific type of journals, and assess students at different grade levels through surveys dealing with students' attitude toward mathematics and by analyzing student journal entries. Students who had used a journal longer showed more depth of reasoning, even if they were at the same grade level, implying that the process of journaling had helped develop these skills. However, this still addresses students' current state, rather than observing actual change in students' skills. Dreyfus noted the absence of studies that study the actual effects of writing tasks on students; he states that most studies "have made little or no attempt to assess changes in student's views of mathematics and their ability to explain and justify. The question how to…help them achieve [reasoning skills] remains open" (1999, p. 106). Therefore, the literature on the use of writing in mathematics is primarily exploratory and descriptive of teacher's experience rather than tied to theory or measuring changes in student learning.

Jurdak & Zein (1998) studied the effects of a different type of journal writing, focusing on student achievement and attitudes in mathematics. The treatment group received the same instruction as the control group, except that they spent 7 to 10 minutes at the end of each class responding to a prompt in their journals. These prompts included both cognitive and affective prompts. Instruments were given as pre- and post-tests to measure students' attitudes and achievement, the latter being subdivided into conceptual understanding, procedural knowledge, problem solving, mathematics school achievement and mathematical communication. The results suggest that journal writing had a positive effect on the areas of conceptual understanding, procedural knowledge and mathematical communication, but not in the

other areas. This study was conducted in both French and English, and found similar benefits regardless of the language used.

Writing as a Means for Assessment

Another approach to research on writing in mathematics is to consider whether writing gives insight into student thinking. Pugalee (1995) found that journal writing about problem solving gave information similar to that gained via think-aloud protocols, which are frequently used to assess problem solving skills, and students performed significantly better on problem solving tasks after another variation of journal writing. This supports the claim that journal writing can help teachers assess students' understanding and higher-level thinking.

Hindrances to Research

There are a number of reasons that research on writing in mathematics tends to be descriptive and theoretical. One of them is the difficulty in measuring the desired student learning, and in fact, disagreement about what that learning should be. Shield and Galbraith (1998) recognized the difficulty in assessing writing and created a method for analyzing students' expository writing. This coding system focused on three areas of the writing: properties of the explanation and justification, aspects of mathematical thinking, and the level of language in terms of generalization and description. Dreyfus notes that, if using writing to develop reasoning, there is a great need for the development of criteria by which teachers can "judge the acceptability of their students' mathematical arguments, and of principles on which the development and examination of such criteria can be based"(1999, p. 106).

Another difficulty is that of communicating clearly about writing tasks. Many aspects of the literature refer to assignments without describing them in detail, or use the same word to refer to very different assignments. For example, a "journal" entry

may refer to writing that students do about their feelings about mathematics and comments to the teacher (Britton in Sterrett, 1990; Rose in Sterrett, 1990), to writing about concepts they are learning (Borasi & Rose, 1989; Burns, 1995), or to writing about problem solving (Patterson, see [Appendix A: 36]). Most of these assignments assume some regularity, but that may be daily, a few times a week or weekly. We know that different assignments can lead to different learning. Dreyfus' concern about standards for judging reasoning also emphasizes how different standards for assessment of a writing task may result in different learning. Detailed descriptions of assignments organized in a framework of writing task variables can set out a research agenda for empirical and theoretical research and make the research more useful to practitioners. Recognizing aspects of the assignment that need to be described and explained is necessary so that teachers can replicate the essential aspects of assignments to promote the desired learning and so that researchers can study which outcomes result from which types of assignments.

We address this second need in this research. We know that it is necessary to carefully define the writing task because research shows that the type of task has a significant effect on the types of learning which result (Applebee, 1987). Therefore, in order to study the learning that results from using writing in mathematics classes, it is necessary to be able to describe and differentiate between assignments in detail.

This research provides a classification to help describe and differentiate writing tasks used in mathematics classes. Categories of variation in writing tasks are identified and examples are given which illustrate the range within each category. This classification introduces a vocabulary that can describe the breadth of variation in secondary mathematics writing activities. This vocabulary is grounded in the practice and language of secondary mathematics teachers to decrease the distance between research and practice: writing activities used in mathematics classrooms were

collected and discussed with the teachers who used them, so that I could hear the language they used to describe the assignments. It is also developed in dialogue with the literature presented here to make connections with studies already completed and to use the theories of how writing and learning interact to suggest variations that may affect learning. With this vocabulary, teachers and researchers can more clearly communicate how writing is used and the resulting student learning. My hope is that this classification will inform and direct research to determine what learning results from different types of writing, and thereby enable teachers to match writing tasks with the student learning they desire.