Math 6410: Enumerative Combinatorics Lecturer: Karola Mészáros

This is a graduate class on enumerative combinatorics.

1 Textbook

Most everything we will do is contained in:

- Enumerative Combinatorics, volumes 1 (referred to as EC1), by R. Stanley.
- An Introduction to Hyperplane Arrangements, by R. Stanley http://www.cis.upenn.edu/~cis610/sp06stanley.pdf

2 Material covered thus far

date topic

- 01/25 Catalan numbers, pattern avoidance, EC1 Sec 1.5, Kostant partition functions & flow polytopes
- 01/30 Permutation statistics and the fundamental bijection EC1 Sec 1.3, 1.4
- 02/01 Generating functions for (various sets of) partitions, set partitions (Stirling numbers of the second kind) EC1 Sec 1.8, 1.9; Tree representations of permutations, counting the number of k-dimensional subspaces of \mathbb{F}_q^n EC1 Secs 1.5, 1.7; Ehrhart polynomial of integer polytopes, magic squares, Birkhoff polytope
- 02/06 Principle of inclusion-exclusion EC1 Secs 2.1, 2.2; Determinantal formulas, Gessel-Viennot Lemma EC1 Sec 2.7
- 02/08 Posets EC1 Secs 3.1, 3.2; order polytopes (Stanley: Two poset polytopes)
- 02/13 Distributive lattices EC1 Secs 3.3, 3.4
- 02/15 Counting chains in distributive lattices, zeta polynomial EC1 Secs 3.5, 3.12 Incidence algebras, Möbius inversion EC1 Secs 3.6, 3.7 order complex of a poset EC1 Sec 3.8
- 02/20 Winter break
- 02/22 integer point enumerators of cones, Ehrhart polynomials, volume, Stanley's nonnegativity theorem
- 02/27 Ehrhart reciprocity, Euler's relation, face lattice of polytopes and its Möbius function
- 03/01 Dehn-Sommerville relations for simplicial polytopes, flag f- and h-vectors, cd-index
- 03/06 Eulerian and simplicial posets and the Dehn-Sommerville relations EC1 Sec 3.17; faces of polytopes (Ziegler: Lectures on Polytopes, Ch 2)
- 03/08 structure theorem for face lattices (Ziegler: Lectures on Polytopes, Ch 2)
- 03/13 Stanley: Two Poset Polytopes (in Discrete & Computational Geometry, 1986)
- 03/15 polytopal complexes, duality of zonotopes and hyperplane arrangements

| 03/20 | hyperplane arrangements (Lecture 1 from http://www.cis.upenn.edu/ \sim cis610/sp06stanley.pdf) |
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| 03/22 | (detour, general overview) Coxeter groups & diagrams; Bruhat order |
| | (source: Björner-Brenti: Combinatorics of Coxeter Groups, Ch 1 & 2) |
| 03/27 | (detour, general overview) Preliminaries on positive and simple roots |
| | (source: Humphrey: Introduction to Lie Alegbras and Representation Theory, Sec 5.4, 5.6, 5.7), |
| 03/29 | (detour, general overview) proof of the Strong Exchange Property |
| | (source: Björner-Brenti: Combinatorics of Coxeter Groups, Ch 1 & 2) |
| 04/10 | hyperplane arrangements (Lecture 2 + Lecture 5 from http://www.cis.upenn.edu/ \sim cis610/sp06stank |
| 04/12 | hyperplane arrangements (Lecture 2 continued); parking functions |
| 04/17-05/08 | permutahedra and other zonotopes corresponding to hyperplane arrangements we've seen |
| | relation to flow and root polytopes |
| | and the many things that there are |

3 What you are expected to do

You are expected to attend and participate in class. There will be problem sets that you will need to solve and hand in. These will appear on the course website http://www.math.cornell.edu/~karola/class641-2018.html.

The classroom is a technology-free zone. If you feel you have a serious reason to use some gadget, please discuss this with me in advance.