MATH 4550: Applicable Geometry

Problem Set 2

Due at 2:54pm before class starts on February 12, 2015

You are allowed to work in groups, but the solutions you hand in should be written by you only. If you work in a group, you must write the names of your collaborators at the top of your assignment. Explain your reasoning to receive full credit. All problems are worth 10 points. You are strongly encouraged to type your solutions in LaTeX.

P1 We say that vectors $\mathbf{x}_1, \ldots, \mathbf{x}_n$ are **affinely independent** if their affine hull (also called affine span) has dimension $n-1$. Let

$$\mathbf{x}_1 = \begin{bmatrix} x_{1,1} \\ \vdots \\ x_{d,1} \end{bmatrix}, \ldots, \mathbf{x}_n = \begin{bmatrix} x_{1,n} \\ \vdots \\ x_{d,n} \end{bmatrix}$$

be n elements of $\mathbb{R}^d$ written as column vectors. Prove that $\mathbf{x}_1, \ldots, \mathbf{x}_n$ are affinely independent if and only if

$$\hat{\mathbf{x}}_1 = \begin{bmatrix} 1 \\ x_{1,1} \\ \vdots \\ x_{d,1} \end{bmatrix}, \ldots, \hat{\mathbf{x}}_n = \begin{bmatrix} 1 \\ x_{1,n} \\ \vdots \\ x_{d,n} \end{bmatrix}$$

are linearly independent in $\mathbb{R}^{d+1}$.

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P2 Let the polytope P be the convex hull of points $(0,0), (1,0)$ and $(0,1)$ in $\mathbb{R}^2$. For each face of P write down a valid inequality for P which defines the face (and explain how you get the face from it). Count the number of faces of P of each dimension.

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P3 Draw down the face poset of a 2-cube. List all chains of the poset and also all maximal chains together with their lengths. Is this poset graded? Is it bounded? Is it a lattice? For each answer give a justification based on the definitions.

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P4 Are the following drawings Hasse diagrams of the face lattice of a polytope? If so, of which polytope? If not, why?