MATH 4550: Applicable Geometry

Problem Set 7

Due at 2:54pm before class starts on April 30, 2015

You are allowed to work in groups, but the solutions you hand in should be written by you only. If you work in a group, you must write the names of your collaborators at the top of your assignment. Explain your reasoning to receive full credit. All problems are worth 10 points. You are strongly encouraged to type your solutions in LaTeX. In any case, please staple your psets!

P1 Show that the dimension of the Minkowski sum of line segments equals the dimension of the linear span of the vectors given by the line segments.

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P2 If Π_n is the permutahedron, that is the convex hull of points in $\mathbb{R}^n$ whose coordinates are the permutations of the numbers $1, 2, \ldots, n$, then prove that the dimension of Π_n is $n-1$, characterize the edges of Π_n and prove that Π_n is simple.

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P3 Given a 3-dimensional polytope such that every two vertices are adjacent, show that it is a tetrahedron.

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P4 A d-polytope P is called *dimensionally ambiguous* if there is a polytope Q of a different dimension which has an isomorphic graph. Show that the d-simplex is dimensionally ambiguous if and only if $d \geq 5$.

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P5 If P is a pointed polyhedron in $\mathbb{R}^3$, show that the graph of all bounded edges is connected. Show that it is not necessarily 2-connected. What happens in higher dimensions?

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P6 (extra credit - unsolved) Is there a 4-polytope whose graph is isomorphic to $G(C_5)$ for the 5-cube C_5?