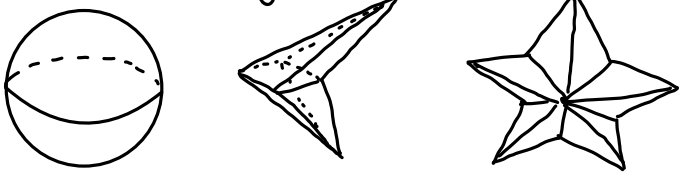
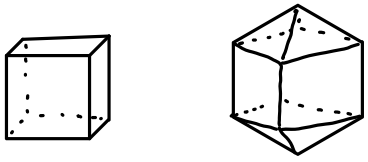
Euler Characteristic of Polytopes

① A polytope is any shape that contains any line between two points inside of it, and has corners and edges.

X:



✓:



② Look at the paper in front of you. Do you recognize any of the shapes? What are they?

③ CRAFT TIME! Make the polytopes. Are the polytopes you made predictions about end up to be what you thought they were? (tetrahedron vs. square pyramid)

④ Now, we are going to count

#faces - #edges + #vertices. BEFORE YOU START, do you expect this number to be the same for all the shapes you made?

Why or why not?

⑤ BEGIN! What do you get?

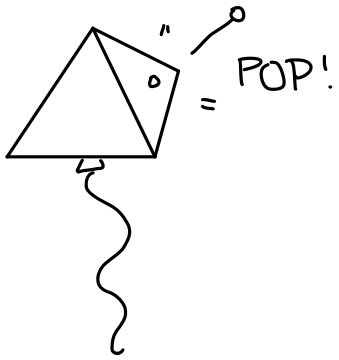
Planning:

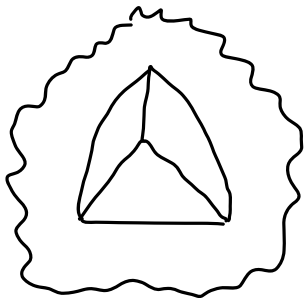
→ Will provide EACH group with a tetrahedron and a cube template. Will also have MANY other templates, distributed among the groups, so they can compare.

- Scissors
- Tape
- Printouts. LOTS of them!

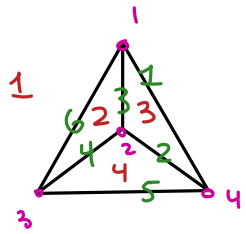
Planar Graphs

① Let's imagine our tetrahedron is a baloon. We "pop" the baloon and stretch it flat.





We will think of the popped face as the "outside" of the following picture.

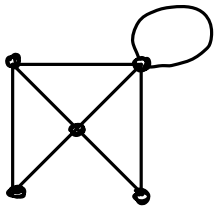


and we count

regions − # lines + # vertices

② Do this for another one of your polytopes (preferably one that's easier to "puncture"). What do you get? Is this what you expect, given our finding from before lunch?

③ Consider a picture with dots and lines, where the lines do not cross!



Count again.

④ Make your own picture. Try this. Compare with other groups. Isn't this weird?

⑤ Why do you think this works?