3 Week Plan

→ This week: Permutations & Triangular numbers

→ Next week: Choosing & Returning to lattices

→ Two weeks from now: Triangulating polygons & Complete graphs (returning to triangular numbers)

Permutations

① Provide 4 sets of 3 distinct objects. These can be, for example, some of our board game pieces.

(Emily: If this is done before lunch, there is a vested interest in not losing the board game pieces)

② Ask them, alongside the board, to count the number of orderings.

③ Have them repeat for 2. Do they have a prediction for 4?

They should think of strategies for counting.

For example last time, with the strings, thinking about if there was a 1 first or a zero first.

④ Use this moment to teach the meaning of factorial.

ASK THEM TO KEEP THEIR PERMUTATIONS ON THE BOARD

⑤ What if we had to put the objects in a circle? Draw a picture on the board as an example.

⑥ Maybe introduce a fourth object.

⑦ What is different about arranging in a circle?

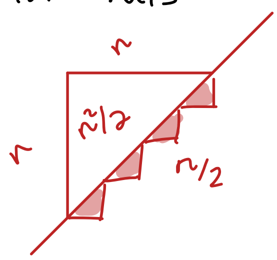
Can you choose orderings that "go with" each circle? Draw lines to connect them in the 3 case.

How many orderings "belong with" each circle?

Triangular Numbers

① How would you draw the sum $1+2+3+4$?
Hint: Think about last time. Partition shapes.

② How would you compute the number of squares in this triangle?



$$\frac{n^2}{2} + \frac{n}{2} = \frac{n(n+1)}{2}$$

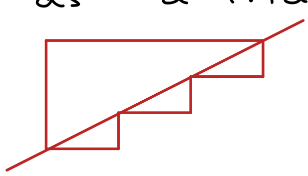
How can you use the fact that these squares make a triangle to come up with an answer to this question?

③ Let's do this for 5!

④ Make a prediction for 6.

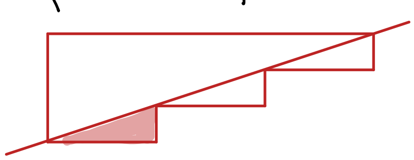
⑤ Can you do this "recipe" for a triangle of squares of ANY SIZE? Test it w/ 6.

⑥ What about the sum $2+4+6+8$, etc? Draw it as a triangle



$$\frac{n(2n)}{2} + n = 2\left(\frac{n^2+n}{2}\right)$$

⑦ Repeat! for $3+6+9+\ldots$ etc.



$$\frac{n(3n)}{2} + \frac{3n}{2} = 3\left(\frac{n^2+n}{2}\right)$$

Do this to keep the fast kids busy. At the end, pull the wool from their eyes.

$$2\left(\frac{n^2+n}{2}\right) = 2(1+2+\ldots+n) = 2+4+\ldots+2n$$

$$3\left(\frac{n^2+n}{2}\right) = 3(1+2+\ldots+n) = 3+6+\ldots+3n$$