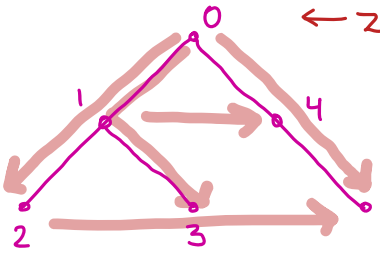
Increasing Trees

① Consider a "root picture" like the one below



← zero at the top

↓ increasing order

→ increasing order

② Here is a game: I provide a list of numbers from 1 to "n".

e.g. 45132

You need to turn this into a "root picture."

Any number in the list goes directly underneath the number that is closest on its left and smaller than it.

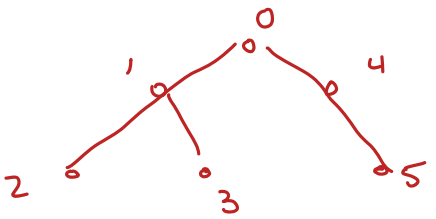
(0) 45132

e.g. 2 goes below 1

3 goes below 1

3 does NOT go below 2, because 2 is NOT to the left of 3.

If it helps, can think of a secret zero here



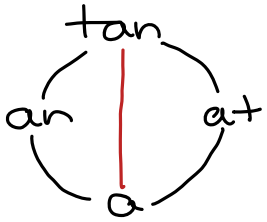
PLAY THE GAME: In groups,
- → one person provide a list of 1,2,3,4
- → everyone figure out the "root picture"
- → take turns giving lists

③ Refresher on factorials and ordering objects. As a group, conjecture/guess how many root pictures there are for 0-7? 0-50? Using what we know about # of lists.

④ With your groups, VERIFY the guess for 0-3. Draw the root pictures and match them with orderings of 1, 2, 3.

Dilworth's Theorem

① Let's take the words tan, an, at, a. I want to make a picture where the longest word is on top, and a line connects words that are contained in one another.



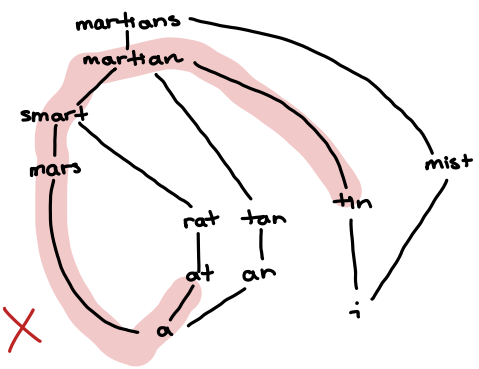
that red line is not helpful, and to avoid cluttering our picture, we do not draw it.

Together, let's all do smart, star, art, at, am, mat.

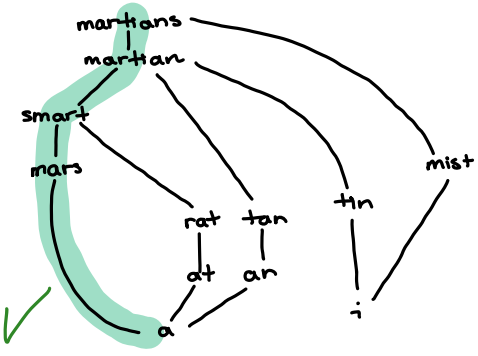
It is important that we do NOT include both art and tar. They have the same letters. (*)

② Keeping the rule (*) in mind, come up with a word picture with your group.

③ A "chain" is a path in your picture (from top to bottom).

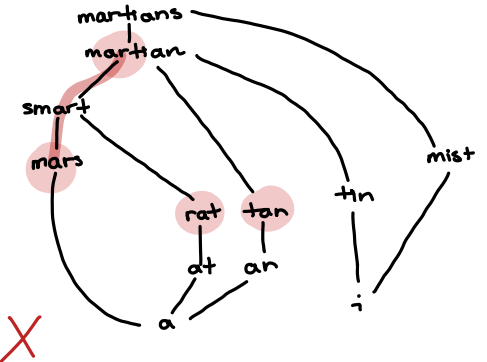


X



✓

③ An "anti-chain" is a group of words where no top → bottom path connects two words.



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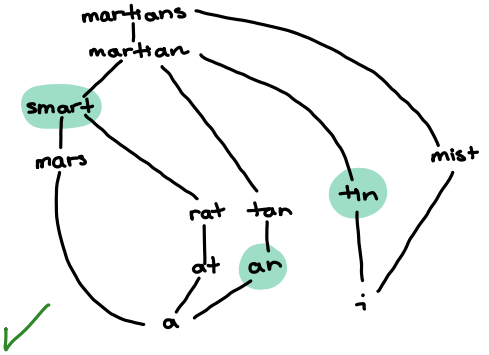


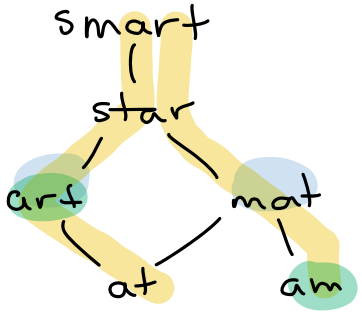
✓

④ Using your pictures, find

(1) what is the largest size of an antichain you can make?

(2) what is the smallest number of chains needed to use every word?

E.g.



2 chains

largest anti-chain has 2 things. (Both the green and blue antichains have 2 things)

⑤ Compare as a big group.