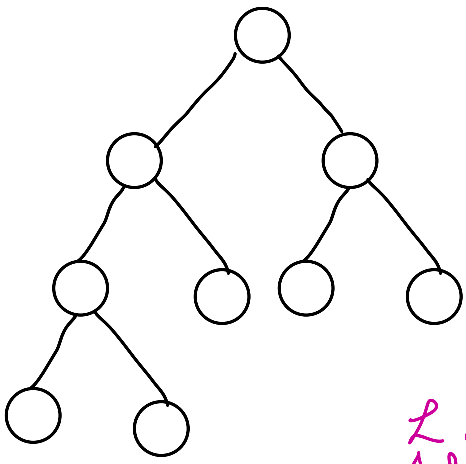
Depth-First Search Game

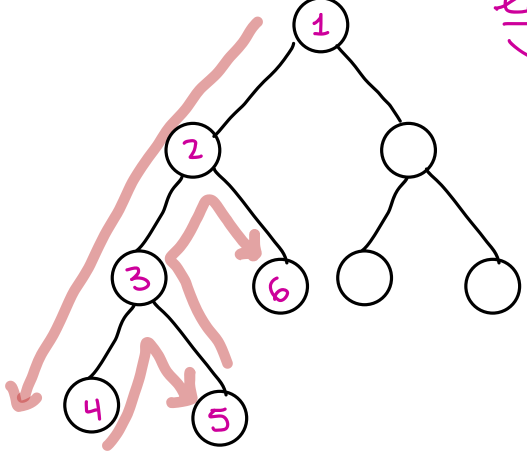
① Consider the "root diagram"



We will describe a recipe for filling the circles with numbers.

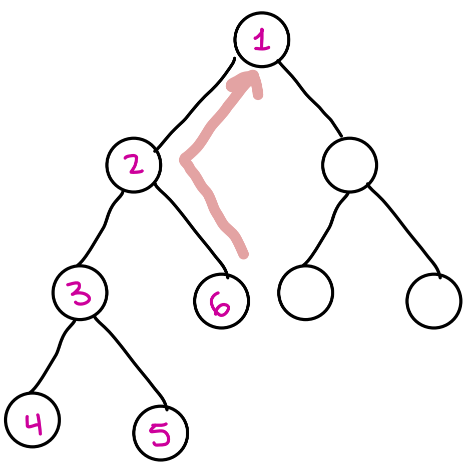
Note: start by drawing 4 copies of this to keep a record the kids can refer to

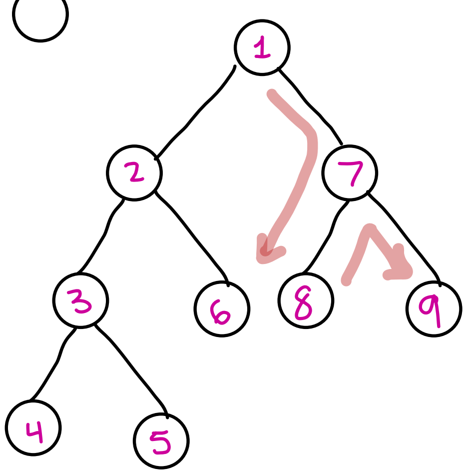
Label the top circle with a 1. Add numbers in increasing order, going <u>left</u> as much as possible until you cannot anymore



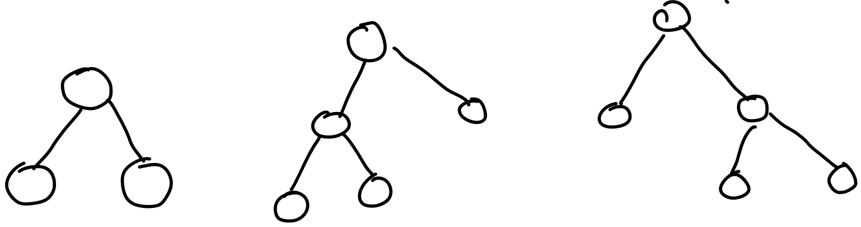
When you reach the bottom, backtrack (draw arrows for clarity)

When you reach the top, repeat, going down first on the right, and sticking to the left.

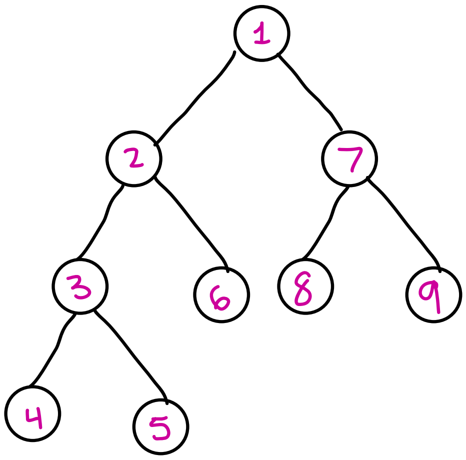




② Try the following (notice the root pictures have two "children" at every node)



③ Come back! We can turn these diagrams with numbers into lists of +1's and -1's. Go in numerical order. L → +1, R → -1



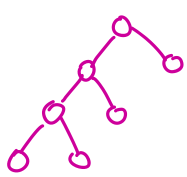
→ +1 +1 +1 -1 -1 -1 +1 -1

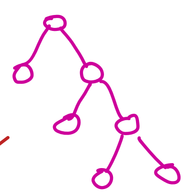
④ What do the numbers in these lists sum to? If we "cut" the lists short, what is true about those sums? (Hint: Are these "cut" sums ever negative?

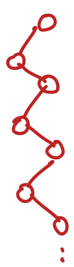
⑤ Try going backwards! Turn

+1 +1 +1 -1 -1 -1 and +1 -1 +1 -1 +1 -1

into root pictures





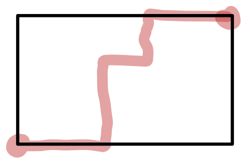


they may be tempted to go

but remind them this is not a "root diagram". Every dot needs two lines. We "pick up where left off.", then must return to top, then go from

Back to Lattice Paths

① Brief refresher on lattice paths.



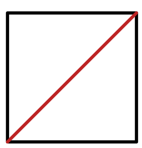
SW corner to NE corner.
only N & E steps.

② How can I turn the lists of +1's and −1's into lattice paths?

(Elena & Prairie: Maybe discuss connection to binary strings with n 1's and m 0's to make lattice path in n×m grid)

③ What do you notice about the dimensions of the grid the path is in?

④ Draw in the "diagonal" of the square

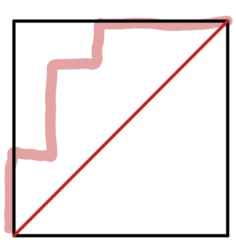


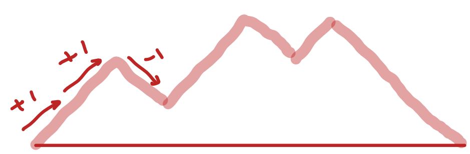
What do you notice about the lattice paths?

⑤ What can we say about the number of lattice paths that do not go (below/above) the diagonal compared to the number of root diagrams from before lunch?

(Consider breaking into 2 steps, with intermediate step being +1 & −1 lists)

⑥ (extra bonus, if time) Tip your lattice paths sideways (may need adjusting, depending on the convention the kids chose, as in paths above or below diagonal).





"cut" sum never goes below zero → "sideways" lattice path height never dips below zero.