MATH 6410: Enumerative Combinatorics

Problem Set 2

Due March 28, 2018

Please LaTeX your answers and email your pdf file to `karola@math.cornell.edu` as well as to `bazse@math.cornell.edu`. You are encouraged to explore the problems in EC1 and hand in anything extra you like.

1. Calculate the Zeta polynomial of the Boolean lattice B_k. Provide a bijective proof.

2. Prove that there exists a 3-polytope with v vertices, e edges and f 2-faces if and only if $v - e + f = 2, v \leq 2f - 4, f \leq 2v - 4$.

3. Are there two distinct face lattices of rank 3 with their respective levels (set of elements of a given rank) equinumerous?

4. Prove that the vertices of the order polytope $\mathcal{O}(P)$ are the characteristic functions of the dual order ideals of the poset P.

5. (extra credit) Characterize the faces of the order polytope $\mathcal{O}(P)$.