Problem 5.5. This is really an easy problem. You just have to check that the supports of $\phi(x)$ and $\phi(2x-k)$ do not intersect for $|k|$ large enough.

Problem 5.7. Let $x = |E|, y = |O|$. Then $x^2 + y^2 = 2$; using the same proof as in Theorem 5.9, we see that $x+y \leq 2$, with the equality achieved only if $x = y = 1$. But $2 = |E + 0| \leq |E| + |O| = x + y$, so $x + y \geq 2 \implies x = y = 1$. Now $E + O = 2, |E| = |O| = 1$ obviously have only one solution $E = O = 1$ (to see that, show that real parts of E and O are equal to 1).

Problem 5.9. (a) Axioms 1 and 4 are trivially satisfied. Axiom 2 is true, because any L^2-function can be approximated closely enough by a continuous function, and any continuous function can be approximated closely enough by a functions in V_n for n large enough, very much like in the proof af Theorem 4.9. Axiom 3 is true as follows: if a function f is in V_n, then its slope on $[0; 2^{-n}]$ stays constant and is equal to right-hand slope at 0. If this slope is α, then L^2-norm of f is at least $\alpha^2 2^{-3n}/3$. If $f \in \cap V_n$, this means that $||f||_{L^2} > \alpha^2 2^{-3n}/3$ for any integer n, so f is not in L^2 - contradiction.

(b) First show that $\{\phi(x-k)\}$ are linearly independent; indeed, otherwise $\sum r_k \phi(x-k) \equiv 0$ for some sequence of b_k, such that $b_l \neq 0$ for some l. But then $f(l) = b_l \neq 0$ - contradiction. Now show that the span of $\{\phi(x-k)\}$ is all V_0. Let $f \in V_0$, $f(k) = a_k$. Let $g(x) := \sum_k a_k \phi(x-k)$. We want to show $f(x) = g(x) \forall x$. Indeed, f and g agree on integers, and for any $k \in \mathbb{Z}$, between k and $k+1$ the graphs of both f and g are straight intervals connecting $(k, f(k))$ and $(k+1, f(k+1))$. So f and g agree on whole $\mathbb{R}$, and $f = \sum_k a_k \phi(x-k)$. This shows $\{\phi(x-k)\}$ is a basis of V_0.

Finding the scaling relations is just an easy exercise in integration.

Problem 5.10. I think that all the guidelines are already given in the problem itself, everything else is just calculation (not the easyest one, though).