Math 424: Homework due on 2/25

1) From the Textbook: Chapter 2: 4, 5.

2) The goal of this problem is to find the Fourier transform of the function f defined by: $f(x) = e^{-x^2}$. Recall that $\int_{-\infty}^{\infty} e^{-x^2}\, dx = \frac{1}{\sqrt{2}}$.

(a) Find $C = \hat{f}(0)$.

(b) Show that $\hat{f}(\gamma)$ is a differentiable function of γ and find $\frac{d\hat{f}}{d\gamma}$ for each $\gamma \in \mathbb{R}$.

(c) Use part b, to show that $\hat{f}(\gamma)$ is a solution to the first order differential equation $Y'(\gamma) + \frac{1}{2}\gamma Y(\gamma) = 0$.

(d) Find the general solution of the previous equation.

(e) Use part a, d, to find $\hat{f}(\gamma)$.

3) Let $f \in L^1(\mathbb{R})$, $a > 0$, $b, c \in \mathbb{R}$ be given. Define the following functions $f_1(x) = a^{1/2} f(ax)$, $f_2(x) = f(x-b)$, and $f_3(x) = e^{2\pi i x c} f(x)$.

(a) Show that f_1, f_2 and f_3 are all in $L^1(\mathbb{R})$ and find the L^1-norm of each of them.

(b) Find the Fourier transform of f_1, f_2 and f_3 in terms of the Fourier transform of f.

(c) Applications. Find the Fourier transforms of f, g and h where $f(x) = e^{-2x^2}$, $g(x) = e^{\pi i x}\chi_{[0,1)}(x)$, and $h(x) = \chi_{[3,4)}(x)$.