**Mathematics 4530**

**Assignment 3, due September 17, 2013**

Read Sections 18 and 19. Then do:

- pp. 111–112: 2, 9(a), [9(b),(c)], 13
- p. 118: 2, 6

Additional problems:

1. Recall that in class I first defined continuity at a point. I then said that a map is *continuous* if it is continuous at every point, and I characterized this in terms of inverse images of open sets. Give a similar characterization of continuity at a point in terms of inverse images. [Hint: The word "mneighborhood" is convenient here.]

2. Let $\overline{\mathbb{N}}$ be the ordered set obtained from the set $\mathbb{N}$ of natural numbers by adjoining a new element $\omega$ that is bigger than every $n \in \mathbb{N}$. (I think of $\omega$ as "infinity".) Give $\overline{\mathbb{N}}$ the order topology.
   (a) Show that the singleton $\{n\}$ for $n \in \mathbb{N}$ is open in $\overline{\mathbb{N}}$, so that this single set forms a mneighborhood base at $n$. (Intuitively, you can't get close to $n$ without being equal to $n$.) Show that the rays $[n, \omega]$ form a mneighborhood base at $\omega$.
   (b) Show that the topological space $\overline{\mathbb{N}}$ has the following property (called *compactness*): If $\mathcal{U}$ is a collection of open sets whose union is $\overline{\mathbb{N}}$, then $\overline{\mathbb{N}}$ is in fact the union of finitely many of the sets $U \in \mathcal{U}$.
   (c) Prove that $\overline{\mathbb{N}}$ is homeomorphic to the set of reals $\{1/n \mid n \in \mathbb{N}\} \cup \{0\}$.