Mathematics 4530

Assignment 6, due October 8, 2013

Read Sections 24, 26, and 27. Then do:

- pp. 157–159: 1, [2], 8
- pp. 170–171: 1, 5
- p. 177: 2

Additional problems:

1. Consider a nested sequence $K_0 \supseteq K_1 \supseteq \cdots$ of subsets of a space X, and let $K := \bigcap_n K_n$. The purpose of this exercise is to find hypotheses under which one can prove the following assertion: $(*)$ If U is a neighborhood of K, then $U \supseteq K_n$ for some n.
   (a) Prove $(*)$ if X is compact and each K_n is closed.
   (b) Prove $(*)$ if X is Hausdorff and each K_n is compact. [Hint: You can deduce this from (a).]

2. If $(U_\alpha)_{\alpha \in J}$ is an open cover of $[0,1]$, show that there is a partition
$$0 = t_0 < t_1 < \cdots < t_n = 1$$
of $[0,1]$ such that each subinterval $[t_{k-1}, t_k]$ is contained in some U_α.

3. Let U be a connected open subset of $\mathbb{R}^n$. Given any two points $x, y \in U$, show that there is a finite sequence
$$x = x_0, x_1, \ldots, x_k = y$$
such that for each pair of consecutive points x_{i-1}, x_i, there is an open ball contained in U that contains the two points. [Hint: Choose a path in U connecting x to y, cover it by small open balls, and consider the induced open cover of the unit interval. Alternatively, imitate the proof that U is path-connected.]