Mathematics 4530

Assignment 7, due Thursday, October 17, 2013

This assignment is shorter than usual because of Fall break and the upcoming prelim.

Read Section 28 and the first four sections of the handout on Tychonoff's theorem. Then do:

- p. 171: 7, 8 [Alternative hint: Consider the projection map $G_f \to X$.]
- p. 181: 6

Additional problems:

1. Recall that the *diameter* of a bounded metric space X is the supremum of the distances $d(x, y)$ for $x, y \in X$. If X is compact, show that there are points x, y such that $d(x, y)$ is equal to the diameter.
2. (e.c.) Prove that every compact metrizable space is homeomorphic to a subspace of the Hilbert cube. [Hint: First prove that such a space X is separable, i.e., that it has a countable dense subset. Now use a metric to construct countably many functions $f_i \colon X \to \mathbb{R}$ that give a 1–1 map from X to a countable product of closed intervals.]