Mathematics 4530 **Prelim 1** **October 22, 2013**

Do any 4 of the following 6 problems, and show clearly which 4 you have chosen. You may do additional problems for fun and a tiny amount of extra credit. Your proofs may use any result proved in class, in the assigned reading, or in the homework, unless you are explicitly asked to prove such a result.

1. Recall that a metric space X is said to be *bounded* if there is a number M such that $d(x, y) \leq M$ for all $x, y \in X$. If X is a finite union of balls, show that X is bounded.

2. Define each of the following terms; verbosity is not necessary.
(a) *topology* on a set X
(b) *topological space*
(c) *subspace topology*
(d) *product topology* (on an arbitrary product, possibly infinite)
(e) *connected* topological space
(f) *compact* topological space

3. Let X be a topological space. Given $B \subseteq A \subseteq X$, topologize A as a subspace of X, and then topologize B as a subspace of A. Show that the resulting topology on B is the same as the topology that B inherits as a subspace of X.

4. Let $\{A_\alpha\}_{\alpha \in J}$ be a family of subsets of a metric space X. Define $f_\alpha \colon X \to \mathbb{R}$ by

$$f_\alpha(x) := d(x, A_\alpha),$$

and let $f \colon X \to \mathbb{R}^J$ be the map with components f_α, where $\mathbb{R}^J := \prod_{\alpha \in J} \mathbb{R}$ is given the product topology. Is f continuous? Justify your answer.

5. Give a proof or counterexample for each of the following statements.
(a) If A and B are connected subspaces of a topological space and $A \cap B \neq \emptyset$, then $A \cup B$ is connected.
(b) If A and B are connected subspaces of a topological space, then $A \cap B$ is connected.

6. Give a proof or counterexample for each of the following statements.
(a) If A and B are compact subspaces of a topological space, then $A \cup B$ is compact.
(b) If A and B are compact subspaces of a Hausdorff space, then $A \cap B$ is compact.