Very Brief Solutions to Final Exam

1. (a) False.
(b) True.
(c) False.
(d) False.

2. See the take-home prelim.

3. (a) Reflexivity and symmetry are obvious. Transitivity follows from the fact that the union of two connected sets with nonempty intersection is connected.
(b) The connected component containing x is the union of all the connected sets containing x.

4. See Munkres.

5. The point at infinity has a countable neighborhood base.

6. (a) See Munkres.
(b) See Munkres.
(c) See Munkres.
(d) The set $A := \{x_1, x_2, \dots\}$ is closed and has the discrete topology. Apply the Tietze extension theorem to the continuous map $f\colon A \to \mathbb{R}$ given by $f(x_n) := n$.

7. (a) See Munkres.
(b) p is an open surjection, so it suffices to prove that it's injective. If $p(\tilde{x}_1) = p(\tilde{x}_2)$, choose a path $\tilde{\alpha}$ from $\tilde{x}_1$ to $\tilde{x}_2$. Its image $\alpha := p(\tilde{\alpha})$ is path homotopic to a constant path, whose lifts are closed, so the lift $\tilde{\alpha}$ is also closed. Thus $\tilde{x}_1 = \tilde{x}_2$.

8. (a) See Munkres.
(b) See additional problem 1 on assignment 13. Or consider a rose with three petals.