

6.2. Identification via invariants

We computed a list of distinguishing invariants for all encoded groups in the library, except those of orders 512 and 1536. This list of invariants is stored in a compressed format and is 41 MB in size. It provides a very efficient approach to identify an encoded group in the library: we compute certain invariants for the given group and locate them in the stored list.

We use a labelled tree to refine our search. The root node is the set of all groups in the library. Each depth level corresponds to a new invariant; the edges linking children to a parent are labelled by different values of this invariant. Each child node is the subset of groups which take this value for the invariant.

Hence, at each level of the tree, we have a partition of all of the groups and by proceeding downwards, this partition is refined. The leaves usually correspond to single groups; in a few cases they correspond to small sets of groups. (In the latter case we use random isomorphism testing to distinguish among the groups in such a set.) Of course, leaves may be at different levels of the tree.

The invariants at the first two levels of the tree are the group order and the abelian invariants of its derived series. If the group is soluble, we next use invariants of a *special* CGS (Eick, [20]); otherwise we use abelian invariants of its centre. At subsequent levels we consider invariants based on conjugacy classes of elements. We partition the conjugacy classes of a group by element order and class length. We use a 3-tuple to identify each subset of the partition: the number of classes it contains, their representative order, and their length. Then we use power-maps and other mappings related to conjugacy classes to split up the given partition of conjugacy classes. Thus we obtain more refined invariants which we use at subsequent levels.

The algorithm is described in Besche and Eick [7, 9] and an implementation is available in **GAP**.

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Appendix A. The number of groups of order at most 2000

	+0	+1	+2	+3	+4	+5	+6	+7	+8	+9
0		1	1	1	2	1	2	1	5	2
10	2	1	5	1	2	1	14	1	5	1
20	5	2	2	1	15	2	2	5	4	1
30	4	1	51	1	2	1	14	1	2	2
40	14	1	6	1	4	2	2	1	52	2
50	5	1	5	1	15	2	13	2	2	1
60	13	1	2	4	267	1	4	1	5	1

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	+0	+1	+2	+3	+4	+5	+6	+7	+8	+9
70	4	1	50	1	2	3	4	1	6	1
80	52	15	2	1	15	1	2	1	12	1
90	10	1	4	2	2	1	231	1	5	2
100	16	1	4	1	14	2	2	1	45	1
110	6	2	43	1	6	1	5	4	2	1
120	47	2	2	1	4	5	16	1	2328	2
130	4	1	10	1	2	5	15	1	4	1
140	11	1	2	1	197	1	2	6	5	1
150	13	1	12	2	4	2	18	1	2	1
160	238	1	55	1	5	2	2	1	57	2
170	4	5	4	1	4	2	42	1	2	1
180	37	1	4	2	12	1	6	1	4	13
190	4	1	1543	1	2	2	12	1	10	1
200	52	2	2	2	12	2	2	2	51	1
210	12	1	5	1	2	1	177	1	2	2
220	15	1	6	1	197	6	2	1	15	1
230	4	2	14	1	16	1	4	2	4	1
240	208	1	5	67	5	2	4	1	12	1
250	15	1	46	2	2	1	56092	1	6	1
260	15	2	2	1	39	1	4	1	4	1
270	30	1	54	5	2	4	10	1	2	4
280	40	1	4	1	4	2	4	1	1045	2
290	4	2	5	1	23	1	14	5	2	1
300	49	2	2	1	42	2	10	1	9	2
310	6	1	61	1	2	4	4	1	4	1
320	1640	1	4	1	176	2	2	2	15	1
330	12	1	4	5	2	1	228	1	5	1
340	15	1	18	5	12	1	2	1	12	1
350	10	14	195	1	4	2	5	2	2	1
360	162	2	2	3	11	1	6	1	42	2
370	4	1	15	1	4	7	12	1	60	1
380	11	2	2	1	20169	2	2	4	5	1
390	12	1	44	1	2	1	30	1	2	5
400	221	1	6	1	5	16	6	1	46	1
410	6	1	4	1	10	1	235	2	4	1
420	41	1	2	2	14	2	4	1	4	2
430	4	1	775	1	4	1	5	1	6	1
440	51	13	4	1	18	1	2	1	1396	1
450	34	1	5	2	2	1	54	1	2	5
460	11	1	12	1	51	4	2	1	55	1
470	4	2	12	1	6	2	11	2	2	1
480	1213	1	2	2	12	1	261	1	14	2
490	10	1	12	1	4	4	42	2	4	1
500	56	1	2	1	202	2	6	6	4	1
510	8	1	10494213	15	2	1	15	1	4	1
520	49	1	10	1	4	6	2	1	170	2
530	4	2	9	1	4	1	12	1	2	2
540	119	1	2	2	246	1	24	1	5	4
550	16	1	39	1	2	2	4	1	16	1
560	180	1	2	1	10	1	2	49	12	1
570	12	1	11	1	4	2	8681	1	5	2
580	15	1	6	1	15	4	2	1	66	1

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	+0	+1	+2	+3	+4	+5	+6	+7	+8	+9
590	4	1	51	1	30	1	5	2	4	1
600	205	1	6	4	4	7	4	1	195	3
610	6	1	36	1	2	2	35	1	6	1
620	15	5	2	1	260	15	2	2	5	1
630	32	1	12	2	2	1	12	2	4	2
640	21541	1	4	1	9	2	4	1	757	1
650	10	5	4	1	6	2	53	5	4	1
660	40	1	2	2	12	1	18	1	4	2
670	4	1	1280	1	2	17	16	1	4	1
680	53	1	4	1	51	1	15	2	42	2
690	8	1	5	4	2	1	44	1	2	1
700	36	1	62	1	1387	1	2	1	10	1
710	6	4	15	1	12	2	4	1	2	1
720	840	1	5	2	5	2	13	1	40	504
730	4	1	18	1	2	6	195	2	10	1
740	15	5	4	1	54	1	2	2	11	1
750	39	1	42	1	4	2	189	1	2	2
760	39	1	6	1	4	2	2	1	1090235	1
770	12	1	5	1	16	4	15	5	2	1
780	53	1	4	5	172	1	4	1	5	1
790	4	2	137	1	2	1	4	1	24	1
800	1211	2	2	1	15	1	4	1	14	1
810	113	1	16	2	4	1	205	1	2	11
820	20	1	4	1	12	5	4	1	30	1
830	4	2	1630	2	6	1	9	13	2	1
840	186	2	2	1	4	2	10	2	51	2
850	10	1	10	1	4	5	12	1	12	1
860	11	2	2	1	4725	1	2	3	9	1
870	8	1	14	4	4	5	18	1	2	1
880	221	1	68	1	15	1	2	1	61	2
890	4	15	4	1	4	1	19349	2	2	1
900	150	1	4	7	15	2	6	1	4	2
910	8	1	222	1	2	4	5	1	30	1
920	39	2	2	1	34	2	2	4	235	1
930	18	2	5	1	2	2	222	1	4	2
940	11	1	6	1	42	13	4	1	15	1
950	10	1	42	1	10	2	4	1	2	1
960	11394	2	4	2	5	1	12	1	42	2
970	4	1	900	1	2	6	51	1	6	2
980	34	5	2	1	46	1	4	2	11	1
990	30	1	196	2	6	1	10	1	2	15
1000	199	1	4	1	4	2	2	1	954	1
1010	6	2	13	1	23	2	12	2	2	1
1020	37	1	4	2	49487365422	4	66	2	5	19
1030	4	1	54	1	4	2	11	1	4	1
1040	231	1	2	1	36	2	2	2	12	1
1050	40	1	4	51	4	2	1028	1	5	1
1060	15	1	10	1	35	2	4	1	12	1
1070	4	4	42	1	4	2	5	1	10	1
1080	583	2	2	6	4	2	6	1	1681	6
1090	4	1	77	1	2	2	15	1	16	1
1100	51	2	4	1	170	1	4	5	5	1

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	+0	+1	+2	+3	+4	+5	+6	+7	+8	+9
1110	12	1	12	2	2	1	46	1	4	2
1120	1092	1	8	1	5	14	2	2	39	1
1130	4	2	4	1	254	1	42	2	2	1
1140	41	1	2	5	39	1	4	1	11	1
1150	10	1	157877	1	2	4	16	1	6	1
1160	49	13	4	1	18	1	4	1	53	1
1170	32	1	5	1	2	2	279	1	4	2
1180	11	1	4	3	235	2	2	1	99	1
1190	8	2	14	1	6	1	11	14	2	1
1200	1040	1	2	1	13	2	16	1	12	5
1210	27	1	12	1	2	69	1387	1	16	1
1220	20	2	4	1	164	4	2	2	4	1
1230	12	1	153	2	2	1	15	1	2	2
1240	51	1	30	1	4	1	4	1	1460	1
1250	55	4	5	1	12	2	14	1	4	1
1260	131	1	2	2	42	3	6	1	5	5
1270	4	1	44	1	10	3	11	1	10	1
1280	1116461	5	2	1	10	1	2	4	35	1
1290	12	1	11	1	2	1	3609	1	4	2
1300	50	1	24	1	12	2	2	1	18	1
1310	6	2	244	1	18	1	9	2	2	1
1320	181	1	2	51	4	2	12	1	42	1
1330	8	5	61	1	4	1	12	1	6	1
1340	11	2	4	1	11720	1	2	1	5	1
1350	112	1	52	1	2	2	12	1	4	4
1360	245	1	4	1	9	5	2	1	211	2
1370	4	2	38	1	6	15	195	15	6	2
1380	29	1	2	1	14	1	32	1	4	2
1390	4	1	198	1	4	8	5	1	4	1
1400	153	1	2	1	227	2	4	5	19324	1
1410	8	1	5	4	4	1	39	1	2	2
1420	15	4	16	1	53	6	4	1	40	1
1430	12	5	12	1	4	2	4	1	2	1
1440	5958	1	4	5	12	2	6	1	14	4
1450	10	1	40	1	2	2	179	1	1798	1
1460	15	2	4	1	61	1	2	5	4	1
1470	46	1	1387	1	6	2	36	2	2	1
1480	49	1	24	1	11	10	2	1	222	1
1490	4	3	5	1	10	1	41	2	4	1
1500	174	1	2	2	195	2	4	1	15	1
1510	6	1	889	1	2	2	4	1	12	2
1520	178	13	2	1	15	4	4	1	12	1
1530	20	1	4	5	4	1	408641062	1	2	60
1540	36	1	4	1	15	2	2	1	46	1
1550	16	1	54	1	24	2	5	2	4	1
1560	221	1	4	1	11	1	30	1	928	2
1570	4	1	10	2	2	13	14	1	4	1
1580	11	2	6	1	697	1	4	3	5	1
1590	8	1	12	5	2	2	64	1	4	2
1600	10281	1	10	1	5	1	4	1	54	1
1610	8	2	11	1	4	1	51	6	2	1
1620	477	1	2	2	56	5	6	1	11	5

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	+0	+1	+2	+3	+4	+5	+6	+7	+8	+9
1630	4	1	1213	1	4	2	5	1	72	1
1640	68	2	2	1	12	1	2	13	42	1
1650	38	1	9	2	2	2	137	1	2	5
1660	11	1	6	1	21507	5	10	1	15	1
1670	4	1	34	2	60	2	4	5	2	1
1680	1005	2	5	2	5	1	4	1	12	1
1690	10	1	30	1	10	1	235	1	6	1
1700	50	309	4	2	39	7	2	1	11	1
1710	36	2	42	2	2	5	40	1	2	2
1720	39	1	12	1	4	3	2	1	47937	1
1730	4	2	5	1	13	1	35	4	4	1
1740	37	1	4	2	51	1	16	1	9	1
1750	30	2	64	1	2	14	4	1	4	1
1760	1285	1	2	1	228	1	2	5	53	1
1770	8	2	4	2	2	4	260	1	6	1
1780	15	1	110	1	12	2	4	1	12	1
1790	4	5	1083553	1	12	1	5	1	4	1
1800	749	1	4	2	11	3	30	1	54	13
1810	6	1	15	2	2	9	12	1	10	1
1820	35	2	2	1	1264	2	4	6	5	1
1830	18	1	14	2	4	1	117	1	2	2
1840	178	1	6	1	5	4	4	1	162	2
1850	10	1	4	1	16	1	1630	2	2	2
1860	56	1	10	15	15	1	4	1	4	2
1870	12	1	1096	1	2	21	9	1	6	1
1880	39	5	2	1	18	1	4	2	195	1
1890	120	1	9	2	2	1	54	1	4	4
1900	36	1	4	1	186	2	2	1	36	1
1910	6	15	12	1	8	1	4	5	4	1
1920	241004	1	5	1	15	4	10	1	15	2
1930	4	1	34	1	2	4	167	1	12	1
1940	15	1	2	1	3973	1	4	1	4	1
1950	40	1	235	11	2	1	15	1	6	1
1960	144	1	18	1	4	2	2	2	203	1
1970	4	15	15	1	12	2	39	1	4	1
1980	120	1	2	2	1388	1	6	1	13	4
1990	4	1	39	1	2	5	4	1	66	1
2000	963									

References

1. C. Archer, The Extension Problem and Classification of Nonsolvable Groups, M.Sc. Thesis, Université Libre de Bruxelles, 1998.
2. G. Bagnara, La composizione dei Gruppi finiti il cui grado é la quinta potenza di un numero primo, *Ann. Mat. Pura Appl.* **1**(3) (1898), 137–228.
3. G. Bagnara, Sopra i gruppi astratti di grado 32, *Ann. Mat. Pura Appl.* **2**(3) (1899), 263–275.
4. D. Baldwin, The Groups of Order 3^n , for $n \leq 6$, B.Sc. Thesis, Australian National University, 1987.
5. P. J. H. Baudet, Groepentheoretische Onderzoekingen, Ph.D. Thesis, Groningen University, 1918.