Cornell University

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Math 6630: Symplectic Geometry, Spring 2018
Problem Set 1

The HW numbers refer to Homeworks in *Lectures on Symplectic Geometry*, A. Cannas da Silva, Lecture Notes in Mathematics, Springer-Verlag, 2008.

1. HW1: 3,4,5,6,7

2. Let F be a finite dimensional vector space, and let $(V = F \oplus F^*, \omega_0)$. Define

$$\omega_0((v,\alpha),(v',\alpha')) = \alpha'(v) - \alpha(v').$$

   a. Show that (V, ω_0) is a symplectic vector space.

   b. Show that if $B\colon F \to F$ is a linear isomorphism and $B^*\colon F^* \to F^*$ the dual map then $B \oplus (B^*)^{-1}\colon V \to V$ is a symplectomorphism.

   c. Show that (V, ω_0) is symplectomorphic to $\mathbb{R}^{2n}$ with the standard symplectic form defined in class.

   d. For $\sigma \in \bigwedge^2 F^*$ define $\omega := \omega_0 + \sigma$ by naturally considering σ as a 2-form on V. Show that the form ω is symplectic and that ω and ω_0 define the same Liouville volume form, that is $\frac{\omega^n}{n!} = \frac{\omega_0^n}{n!}$ where $n = \dim F$.

3. Prove the following Lemma.

   Lemma: Let (V, ω) be a symplectic vector space and let M be a Lagrangian subspace of V. Then, there exists a Lagrangian subspace $L \subset V$ with $M \cap L = \{0\}$.