Cornell University

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Math 6630: Symplectic Geometry, Spring 2018
Problem Set 3

The HW and page numbers refer to Homeworks in *Lectures on Symplectic Geometry*, A. Cannas da Silva, Lecture Notes in Mathematics, Springer-Verlag, 2008.

1. HW5: 1

2. Prove Cartan's magic formula: $\mathcal{L}_v\omega = \iota_v d\omega + d\iota_v\omega$, and the formula $\frac{d}{dt}\rho_t^*\omega = \rho_t^*\mathcal{L}_{v_t}\omega$, where ρ_t is the local isotopy integrating the vector field v_t. See guidance in p.42.

3. Prove the following theorem of Moser:

   Theorem: Let M be a compact oriented manifold and let $\mathcal{L}_0, \mathcal{L}_1$ be two volume forms on M such that
   $$\int_M \mathcal{L}_1 = \int_M \mathcal{L}_2.$$
   Then, there is a smooth family of diffeomorphisms $\phi_t \in \mathrm{Diff}(M)$ such that $\phi_0 = \mathrm{Id}_M$ and $\phi_1^*\mathcal{L}_1 = \mathcal{L}_0$.

   Hint: Take inspiration from the Moser Lemma in symplectic geometry.