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Math 6630: Symplectic Geometry, Spring 2018
Problem Set 4

The HW numbers refer to Homeworks in *Lectures on Symplectic Geometry*, A. Cannas da Silva, Lecture Notes in Mathematics, Springer-Verlag, 2008.

1. The complex torus $(\mathbb{C}^*)^n$ acts on $\mathbb{C}^n$ by coordinate-wise multiplication

$$(\lambda_1, \ldots, \lambda_n)(z_1, \ldots, z_n) = (\lambda_1 z_1, \ldots, \lambda_n z_n).$$

   Is the action proper? What are the orbits? What are the stabilizers? Describe the quotient $\mathbb{C}^n/(\mathbb{C}^*)^n$.

   Describe the orbits, the stabilizers, and the quotient for the action of the compact torus $(S^1)^n \subseteq (\mathbb{C}^*)^n$ that is given by the same formula.

2. Fix a real number α and let $\mathbb{R}$ act on the torus $T^2 = \mathbb{R}^2/\mathbb{Z}^2$ by

$$t \cdot (x, y) = (x + t, y + \alpha t).$$

   Find the orbits and show that these are submanifolds of T^2 if and only if α is rational. (Hint: if α is irrational, all the orbits are dense in T^2).

3. Show that the brackets $[\cdot, \cdot]\colon \mathfrak{g} \times \mathfrak{g} \to \mathfrak{g}$ defined by $[X, Y] := \mathrm{ad}_X(Y)$ are indeed Lie brackets: anti-symmetric and satisfy the Jacobi identity.

4. HW 19: 1