Math 1110 Midterm

July 10, 2015

Name: _____

- Do not open this book until instructed to do so.
- You will have 75 minutes to complete the exam. **SHOW WORK** and/or justification on all problems: even if your final answer is incorrect you may receive partial credit for the reasoning displayed. Books, notes, calculators, cell phones, and other forms of assistance are not to be used during the exam.
- Each problem appears on its own page. Feel free to use the back of the page to continue your work; there are also two extra sheets of paper at the end for scratch work. Please answer each question on its page - the scratch work will not be graded.
- Please adhere to the honor code.

Problem	Score
1	/20
2	/15
3	/20
4	/15
5	/15
6	/15
TOTAL	/100

Signature: _____

1. (15 pts) Compute the following limits, justifying your steps.
   L'Hopital's Rule is NOT allowed.

   (a) $\lim_{x \to \pi} e^{\frac{1}{x} + \cos(x)}$

   (b) $\lim_{x \to -\infty} \dfrac{x+1}{\sqrt{x^2+1}}$

   (c) $\lim_{x \to \pi} \dfrac{\sin(x) - \sin(\pi)}{\pi - x}$
   (Hint: Recall the definition of the derivative of $\sin(x)$.)

2. (20 pts) Compute the following derivatives:

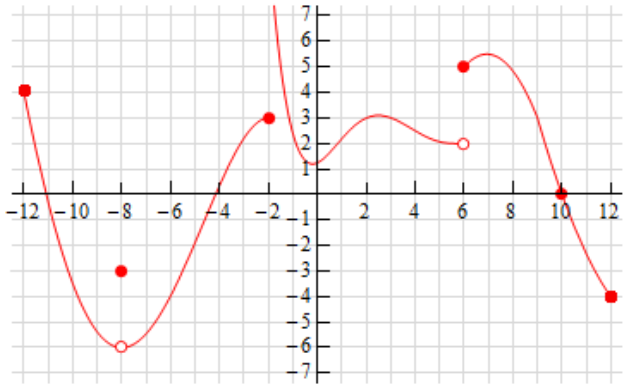
(a) $\dfrac{d}{dx}\left(x^3 e^x\right)$

(b) $\dfrac{d}{d\theta}\left(\tan(2\theta+1) - 3\theta + e^2\right)$

(c) $\dfrac{d}{dy}\left(\dfrac{\sin(y)+1}{y+1}\right)$

(d) $\dfrac{d}{dx}\left(x^{\cos(x)} + \ln(x)\right)$

3. (20 pts) Consider the following graph of the function $f(t)$:



(a) $\lim_{t \to -2-} f(t) =$

(b) $\lim_{t \to -6} f(t) =$

(c) $\lim_{t \to -8} f(t) =$

(d) Estimate the instantaneous rate of change of f at $t = 10$ (don't worry too much about getting it perfect).

(e) Estimate the average rate of change of f on the interval $[2,\ 6]$.

(f) Does the graph f have any vertical asymptotes? If so, what are the equations?

(g) An interval on which f is decreasing is: _____

(h) Is f a 1-to 1 function, and why?

(i) The range of f on the interval [-12, 12] is: _____

(j) Put an x on the t-axis at every point in the domain of f where f is not continuous.

(k) Put a circle $\circ$ on the t-axis at every point in the domain of f where f is continuous but not differentiable.

4. (15 pts) Find the equation of the tangent line to the curve

$$x + \sqrt{xy} = 6$$

at the point $(4, 1)$.

5. (15 pts) Suppose $f(x)$ is a continuous function with domain $[-5, 5]$. Let $g(x) = x$. If $f(-5) = 6$ and $f(5) = -6$, show that there is $c \in [-5,\ 5]$ such that $f(c) = g(c)$.

6. (15 pts) Consider the function

$$f(x) = \left\{ \begin{array}{ll} x^2 \sin(\frac{1}{x}) & x \neq 0 \\ 0 & x = 0 \end{array} \right\}$$

(a) Is f differentiable at $x = 3$, and why or why not?

(b) Find the derivative of f at $x = 0$.

(c) Is f an even function? Odd function? Both? Neither?