August 4, 2014 Name: ______________________________

Math 1110 – FINAL

- Do not open this booklet until instructed to begin.
- You will have $2\frac{1}{2}$ hours to complete the exam. This test has 12 questions, worth a total of 150 points. Please **SHOW ALL YOUR WORK**: even if your final answer is incorrect you may receive partial credit for the reasoning displayed. Books, notes, calculators, cell phones, and other forms of assistance are not to be used during the exam.
- Each problem appears on its own page and the pages are double sided. There are also two extra sheets of paper at the end for scratch work. Please answer each question on its page - the scratch work will not be graded.

PROBLEMS	GRADES
1	________ / 16
2	________ / 12
3	________ / 12
4	________ / 12
5	________ / 10
6	________ / 10
7	________ / 10
8	________ / 16
9	________ / 10
10	________ / 12
11	________ / 12
12	________ / 18
Total	________ / 150

1. **(16 pts) Limits** Evaluate

(a) Suppose that $f(x)$ is an even function with $\lim_{x \to 5} f(x) = 2$, and $g(x)$ is an odd function with $\lim_{x \to 5} g(x) = 3$. What is $\lim_{x \to -5} \frac{7f(x)+1}{g(x)}$?

(b) $\lim_{x \to \infty} (\sqrt{9x^2 - x} - 3x) =$

(c) $\lim_{x \to 0} (\cos x)^{1/x} =$

(d) $\lim_{x \to \pi} \frac{\int_{\pi}^{x} \cos t + 1 \, dt}{(x - \pi)^3} =$

2. **(12 pts) Derivatives**

x	$f(x)$	$f'(x)$	$f''(x)$	$g(x)$	$g'(x)$
-2	6	-1	-10	-4	6
-1	5	-2	3	1	0
0	2	-1	1	4	2
1	0	-4	8	2	-4
2	-1	-3	2	5	3

Let f and g be differentiable functions where f is one-to-one. Use the above table to determine the derivatives of the following functions at the given points. Note that it is important to take the derivatives first before plugging in the values for x.

(a) $h(x) = f^{-1}(x)$ at $x = 2$

(b) $j(x) = \int_{g(x)}^{5} (t^2 + \ln t)\, dt$ at $x = 1$

(c) $k(x) = \frac{2^x}{f'(x)}$ at $x = 0$

(d) $p(x) = \arctan(g(x))$ at $x = -2$

3. **(12 pts)** Let $f(x)$ be an ***even*** continuous function and $g(x)$ be an ***odd*** continuous function with

$$\int_{1}^{5} f(x)\,dx = -2, \qquad \int_{-1}^{5} f(x)\,dx = 8, \qquad \int_{1}^{5} g(x)\,dx = 7$$

Find

(a) $\displaystyle\int_{-5}^{-1} g(x) + 3f(x)\,dx$

(b) $\displaystyle\int_{-1}^{0} f(x)\,dx$

(c) $\displaystyle\int_{5}^{-1} g(x)\,dx$

4. **(12 pts)** Evaluate the following

(a) $\displaystyle\int_0^{\pi/4} \sin^2(2\theta)\cos(2\theta)\ d\theta$

(b) $\displaystyle\int x\sqrt{1+x}\ dx$

(c) $\displaystyle\int_0^2 5+\sqrt{4-x^2}\ dx$

5. **(10 pts)** The acceleration of a particle (moving along a straight line) is given by $a(t) = \sin(\frac{t}{2}) + 6t$. Find the position of the particle at time $t = \pi$ if the initial velocity was $v(0) = 5$ and the initial position was $s(0) = 1$. (*You do not need to simplify your final answer.*)

6. **(10 pts)** Let

$$f(x) = \begin{cases} x^2, & x \leq 2 \\ mx + b, & x > 2 \end{cases}$$

Find the values of m and b that make f differentiable everywhere.
(*Remember to show all your work!*)

7. **(10 pts) Graphing:** Consider the function

$$f(x) = 4x^3 - 24x^2 + 36x = 4x(x-3)^2$$

(a) Find the intervals on which f is increasing and the intervals on which f is decreasing.

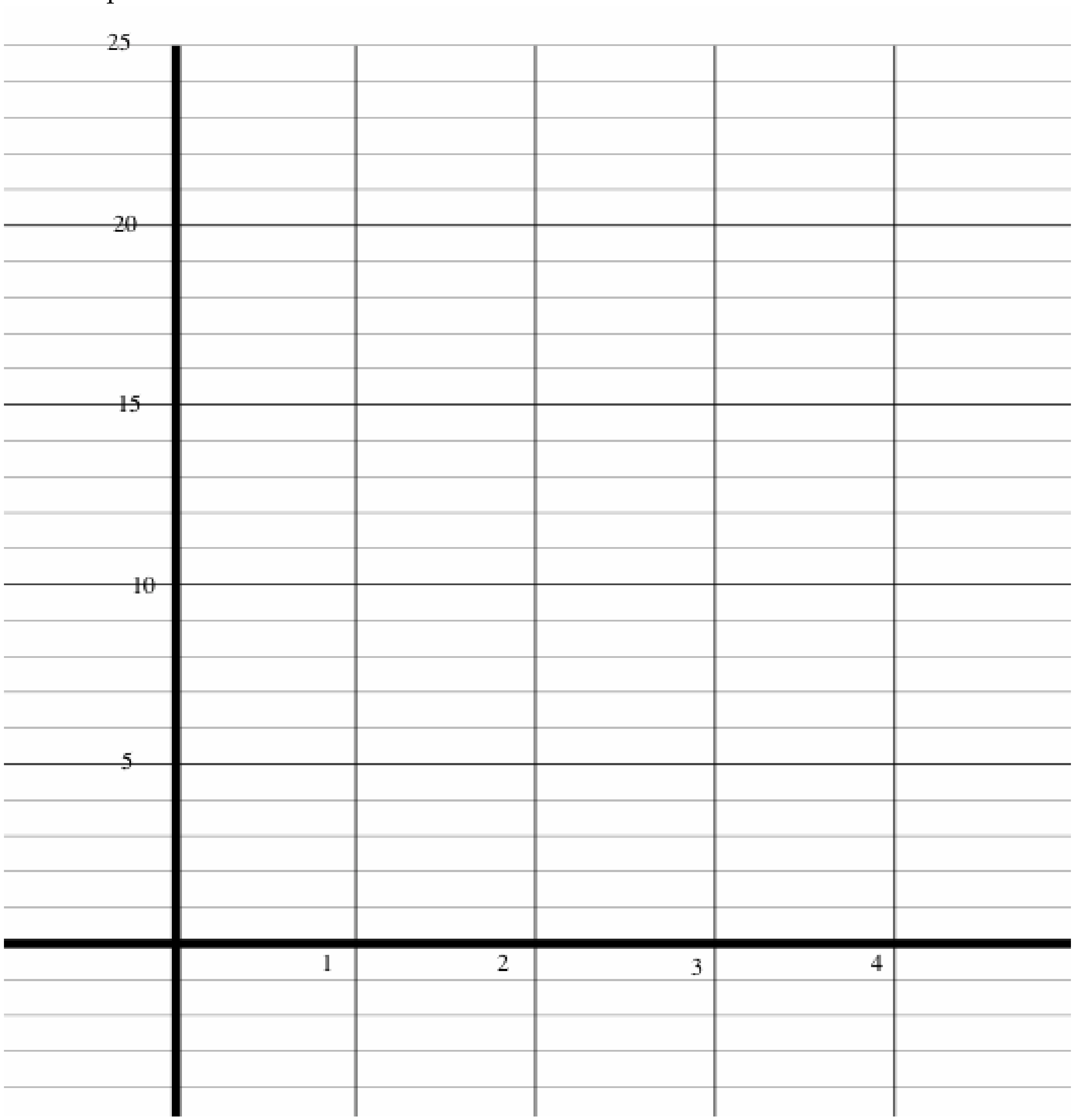
(b) Find the intervals on which f is concave up and the intervals on which f is concave down.

(c) Find any local maximums and local minimums and determine which, if any, are global extreme values.

(d) Using what you have calculated in the first four parts, please graph the function

$$f(x) = 4x^3 - 24x^2 + 36x$$

on the axes given below. Label any local extreme points, as well as all inflection points.



8. **(16 pts)** Determine whether the following statements are (always) true or (at least sometimes) false, and circle your response. Please give a brief explanation (in complete sentences!) - a reason why it's true, or an example of where it fails.

(a) If $f(x)$ is differentiable with $0 \leq f'(x) \leq 3$ for all x, then $f(3) \leq f(0) + 9$.

True	False

(b) If the area of a circle is changing at a constant rate, then so does its radius.

True	False

(c) Let f be a function such that $\lim_{x \to 1^+} f(x) = 3$ and suppose f is differentiable at $x = 1$. Then $f(1)$ must equal 3.

True	False

(d) If a marathoner ran the 26.2 mi NYC Marathon in 2 hours, then at least twice the marathoner was running at exactly 11 mph (assuming the initial and final speeds are zero).

True	False

9. **(10 pts)** Show that the equation $x^5 + 2x^3 + 5x - 3 = 0$ has exactly one solution.

10. **(12 pts)** TCAT Bus passes are sold to students for 40 dollars each. Five hundred students are willing to buy them at that price. For every 2 dollar increase in price, there are 10 fewer students willing to buy the pass. What selling price will produce the maximum revenue and what will the maximum revenue be?

11. **(12 pts) Area:**

(a) Draw the region enclosed by the curves $y^2 = 4x$ and $y = 2x - 4$. Make sure to label your axes and the intercepts.

(b) Find the area of the region enclosed by the curves $y^2 = 4x$ and $y = 2x - 4$ by integrating with respect to y.

(c) Find the area of the region enclosed by the curves $y^2 = 4x$ and $y = 2x - 4$ by integrating with respect to x.

12. **(18 pts) Riemann sums:** Consider the given integral

$$\int_1^3 x^2 + 1 \, dx$$

(a) Find an approximation for the integral using the area of four rectangles with base of equal length and height that corresponds to the right endpoint of the base.

(b) Find an approximation for the integral using the area of n rectangles with base of equal length and height that corresponds to the right endpoint of the base. Leave it in sigma notation.

(c) Find a closed form for the Riemann sum you found in (b). Note that

$$\sum_{k=1}^{n} k = \frac{n(n+1)}{2} \qquad\qquad \sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$$

(d) Use the answer you found in (c) to determine the actual value for the integral.

(e) Use the Fundamental Theorem of Calculus to determine the actual value for the integral.

SCRATCH WORK

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