Midterm Exam

Math 1110, July 13, 2012

NAME: INSTRUCTOR:

Problem 1. Find the indicated derivatives.

1.a) *(5 points)* Let $y(x) = x^5 + 2.5x + \pi^2$. Find $y'(x)$.

1.b) *(6 points.)* $\frac{d}{d\theta} \sin(\cos(\theta))$

1.c) *(10 points.)* Let $f(x) = 2\sin(2x) + e^{-x} + x$. Find $f^{(n)}(x)$, for $n = 1, 2, 3, 4$ and 5. (Recall $f^{(n)}$ denotes the n^{th} derivative of f.)

Problem 2. Consider the function

$$f(x) = \begin{cases} \sin(x) & \text{if } x < 0 \\ ax + b & \text{if } 0 \le x < 1 \\ \frac{1}{2}x^2 + \frac{1}{2} & \text{if } 1 \le x, \end{cases}$$

where a and b are real numbers.

2.a) *(8 points)* For what value of a and what value of b is f continuous at every point in its domain?

2.b) *(8 points)* For the values of a and b found in 2.a), at which values of x is f differentiable?

2.c) *(8 points)* For the values of a and b found in 2.a), write an expression for $f'(x)$ on the domain found in part 2.b)

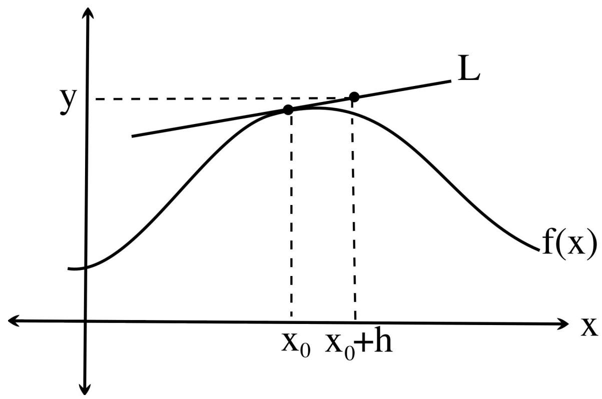
Problem 3. *(9 points)* Let $g(x) = 2x^3 + 3x^2 - 12x + 1$. Find all points $(x, g(x))$ at which the tangent to g is horizontal. Write equations for all such tangent lines.

Problem 4

4.a) *(10 points)* Let $f(x) = \sqrt{x}$, with domain $[0, \infty)$. Use the definition of the derivative to compute $f'(x)$.

4.b) *(4 points)* Are there any points in the domain of f which are not in the domain of f' ? If so, which points?

Problem 5. *(10 points)*



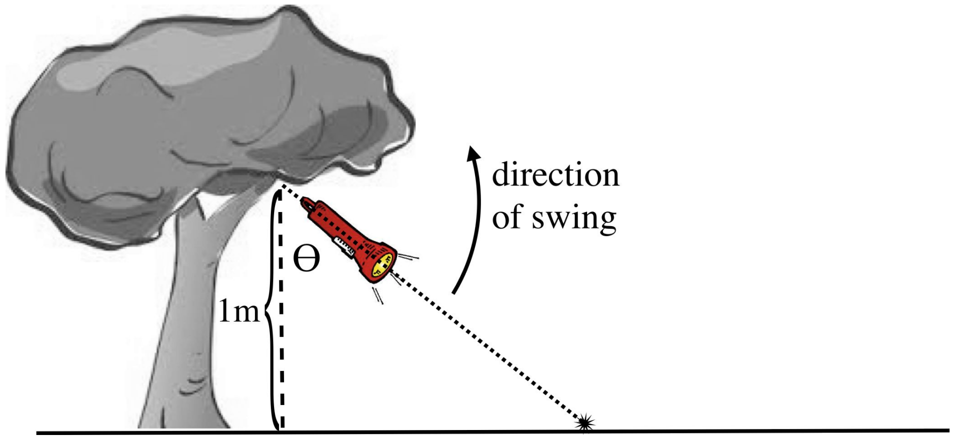
The figure above shows the graph of a function f; you do not know the equation for f. You do know the values of $f(x_0)$ and $f'(x_0)$. For some small h, the point $(x_0 + h, y)$ is on the line L, the tangent to the graph of f at x_0. Find an expression for y.

Problem 6. *(10 points)* Let $f(x) = 2x^2 + x$. Let $x_0 = -1$ and let $\epsilon = \frac{1}{4}$. Find a real number $\delta > 0$ such that

$$|x - x_0| < \delta \text{ implies } |f(x) - f(x_0)| < \epsilon.$$

Show that your δ works.

Problem 7 *(12 points)*



As in the figure above, you are sitting in a tree and swinging a flashlight in the counterclockwise direction. The height of the end of the flashlight (which does not move as you swing) is 1 meter, and you swing the flashlight at an angular rate of 2π radians per second. Let θ be the angle between the downwards direction and the direction your flashlight is pointing. When $\theta = \pi/4$, how fast is the beam traveling over the ground?