**2005 Cornell Summer School in Probability**

| | |
|---|---|
| | **Monday, July 11** (251 Malott) |
| 08:45—09:00 | Welcome and introduction |
| 09:00—10:30 | Durrett, Lecture 1 |
| 10:50—12:20 | Lyons, Lecture 1 |
| 15:30—15:55 | F. Sobieczky, Random walks on the finite components of random partial graphs of transitive graphs |
| 16:30—16:55 | J. Mairesse, Random walks on groups, braids, and Tetris heaps |
| 17:00—17:25 | C. Mueller, The speed of a random traveling wave |

| | |
|---|---|
| | **Tuesday, July 12** (251 Malott) |
| 09:00—10:30 | Le Gall, Lecture 1 |
| 10:50—12:20 | Durrett, Lecture 2 |

| | |
|---|---|
| | Afternoon Session I (251 Malott) |
| 15:30—15:55 | X. Zhang, Expected length of minimum spanning tree |
| 16:30—16:55 | M. Hosseini, A sharp inequality for the first exit time of Brownian motion |
| 17:00—17:25 | P. Kim, Relative Fatou's theorem for $(-\Delta)^{\alpha/2}$-harmonic functions in bounded $\kappa$-fat open sets |

| | |
|---|---|
| | Afternoon Session II (253 Malott) |
| 15:30—15:55 | N. Lanchier, Epidemics with recovery on a lattice of social clusters) |
| 16:30—16:55 | R. Schinazi, Mathematical models for virus dynamics) |
| 17:00—17:25 | D. Welch, Building and fitting models of coupled host-virus genealogies) |

| | |
|---|---|
| | **Wednesday July 13** (251 Malott) |
| 08:45—09:00 | Lyons II |
| 10:50—12:20 | Le Gall II |
| 15:30—15:55 | T. Melcher , Malliavin calculus and heat kernel inequalities |
| 16:30—16:55 | F. Zhang , Harmonic functions of subordinate killed Brownian motion in exterior domain |
| 17:00—17:25 | X. Zhou, Exit problems for reflected spectrally negative Levy processes |

| | |
|---|---|
| | **Thursday, July 14** (251 Malott) |
| 08:45—09:00 | Durrett, Lecture 3 |
| 10:50—12:20 | Lyons, Lecture 3 |
| 15:30—15:55 | S. Blachere , Internal DLA on discrete groups having exponential growth |
| 16:30—16:55 | M. Merle, Estimates for the probability of hitting far points for the voter model |
| 17:00—17:25 | S. Cioba, Eigenvalues, Ramanujan graphs, and expanders |

| | |
|---|---|
| | **Friday, July 15** (251 Malott) |
| 10:00—10:25 | N. Shank, Nearest neighbor graphs on Cantor sets |
| 10:50—12:20 | J.-F. Le Gall, Lecture 3 |
| 15:30—15:55 | S. Brofferio, Some properties of the lamplighter's walks |
| 16:30—16:55 | D. Bertacchi, Asymptotic behaviour of the simple random walk on the 2-dimensional comb |
| 17:00—17:25 | F. Zucca, The discrete integral maximum principle and estimates of transition probabilities |

| | |
|---|---|
| | **Saturday, July 16** (Room TBA) |
| 09:50—10:15 | G. Lawler, Laplacian random walk |
| 10:20—10:55 | A. Lambert, The logistic branching process, coalescence with fragmentation, and fixation of mutant alleles |
| 11:00—11:25 | O. Zhao, Laws of the iterated logarithm for functional of Markov chains |
| 11:30—11:55 | A. Rudas, Random trees and general branching processes |

| | **Monday, July 18** (Room TBA) |
|---|---|
| 09:00—10:30 | R. Durrett, Lecture 4 |
| 10:50—12:20 | R. Lyons, Lecture 4 |

| | Afternoon Session I (Room TBA) |
|---|---|
| 15:30—15:55 | B. D'Auria, Limit flows in queueing networks |
| 16:30—16:55 | C. Sauer, Stochastic networks with unreliable nodes |
| 17:00—17:25 | C. Gromoll, Measure valued processes and queues |

| | Afternoon Session II (Room TBA) |
|---|---|
| 15:30—15:55 | G. Kordzhakia, A predator-prey model on homogeneous trees |
| 16:30—16:55 | M. Krishnapur, Two-dimensional determinantal processes |
| 17:00—17:25 | Y. Atchade, A limit result for adaptive Markov chains |

| | **Tuesday, July 19** (Room TBA) |
|---|---|
| 09:00—10:30 | J.F. Le Gall, Lecture 4 |
| 10:50—12:20 | R. Durrett, Lecture 5 |
| 15:30—15:55 | O. Angel , Multi-type exclusion processes |
| 16:30—16:55 | V. Sood, Voter model on heterogeneous graphs |
| 17:00—17:25 | A. Roitershtein, LLN and CLT for multi-type branching processes |
| | iwith immigration in RE and for RWRE in a strip |

| | **Wednesday, July 20** (Room TBA) |
|---|---|
| 09:00—10:30 | R. Lyons, Lecture 5 |
| 10:50–11:15 | L. Dicker, Partitioning sites of $\mathbb{Z}^3$ according to which random walk gets there first |
| 11:20—11:45 | G. Szabo, Geometry and degeneracy of minimum spanning trees on random networks |
| 11:50—12:15 | A. Chaintreau, First passage percolation, in lattice and beyond, and its application to data networks |

| | **Thursday, July 21** (Room TBA) |
|---|---|
| 09:00—10:30 | J.F. Le Gall, Lecture 5 |
| 10:50—12:20 | R. Durrett, Lecture 6 |
| 15:30—15:55 | L. Jones, Number variance saturation |
| 16:30—16:55 | A. Begyn , Quadratic variations for Gaussian processes for irregular subdivisions |
| 17:00—17:25 | S. Chatterjee, Concentration of Haar measures, with an application to random matrices |
| 17:30—17:55 | P. Otto, Limit theorems for a class of mean-field models with varying temperature |

| | **Friday, July 22** (Room TBA) |
|---|---|
| 09:00—10:30 | R. Lyons, Lecture 6 |
| 10:50—12:20 | J.-F. Le Gall, Lecture 6 |
| 15:30—15:55 | M. Lladser, Multiple pattern frequencies for Markovian s sequences |
| 16:30—16:55 | G. Kerns, Exchangeability and de Finetti's Theorem |
| 17:00—17:25 | C. Cotar, Some new results on edge reinforced random walk |

**ABSTRACTS FOR PARTICIPANT TALKS**

Cornell Summer School in Probability, 2005

Omer Angel, Mutli-type exclusion processes

Yves Atchade, A limit result for adaptive Markov Chains

We consider a class of random processes with long range dependence that arise in adaptive Markov Chain Monte Carlo methodology. We use a coupling argument to obtain a convergence and a (bound on) rate of convergence for such processes.

Joint with Christophe Andrieu, Bristol, UK.

Bernardo D'Auria, Limits flows in queueing networks

Arnaud Begyn, Quadratic variations for Gaussian processes for irregular subdivisions

In the theory of fractional processes one uses the second order quadratic variation, in order to estimate the regularity and the self-similarity index of the process. We study the convergence of the second order quadratic variation when it is computed along irregular subdivisions, and we show that the limit is related to the process and the strucuture of the subdivisions too. We study the example of the time-space deformed fractional Brownian motion, which include the fractional Ornstein-Uhlenbeck process.

Daniela Bertacchi, Asymptotic behaviour of the simple random walk on the 2-dimensional comb

The 2-comb is one of the simplest examples of inhomogeneous graphs, as it is obtained from $\mathbb{Z}^2$ by deleting all horizontal lines off the x-axis. We analyze the differences between the horizontal and the vertical component of the simple random walk on this graph. In particular we evaluate by combinatorial methods the asymptotic behaviour of the expected value of the distance from the origin, the maximal deviation and the maximal span in $n$ steps, proving that for all these quantities the order is $n^{1/4}$ for the horizontal projection and $n^{1/2}$ for the vertical one (the exact constants are determined). Then it is natural to rescale the two projections of the random walk dividing by $n^{1/4}$ and $n^{1/2}$ the horizontal and vertical ones, respectively. The limit process is obtained. As a corollary of the estimate of the expected value of the maximal deviation, the walk dimension of the 2-comb is determined, showing that the Einstein relation between the fractal, spectral and walk dimensions

Sebastien Blachere, Internal Diffusion Limited Aggregation on discrete groups having exponential growth

The Internal Diffusion Limited Aggregation has been introduced by Diaconis and Fulton in 1991. It is a growth model defined on an infinite set and asociated to a Markov chain on this set. We focus here on sets which are Cayley graphs of finitely generated groups with exponential growth. We prove a shape theorem for the Internal DLA on such groups associated to symmetric random walks. For that purpose, we introduce a new distance associated to the Green function, which happens to have some interesting properties.

Sara Brofferio, Some properties of the lamplighter's walks

Consider a lamplighter that randomly walks along a street and that randomly switches on or off the lampposts that stand at every crossing. In this way, one constructs a random process on the set of all possible configurations of 0 and 1 on the integer line (the lampposts) times the integer line it self (the position of the lamplighter). In the last years mathematicians have been interested in such stimulating model. I would like to presents two joint works with W.Woess, on this subject. We

associate to the random walk of the lamplighter a random walk on a suitable graph, called Diestel-Leader graph, that is the horocyclic product of two trees. Thank to this approach, we obtain a deeper understanding of the geometry of the lamplighter walk and we are able to deduce the precise asymptotic behavior of the Green kernel and a complete description of the harmonic functions.

Augustin Chaintreau, First passage percolation, in lattice and beyond, and its application to data networks.

The success of the decentralized control of data networks as implemented by the TCP/IP protocol opened a number of exciting theoretical problems related to the scalability and performance of communication systems. In this talk I would like to briefly introduce this field and present one of them, that is related to the interaction of large number of connections inside a simple reliable group communication scheme. The speed of data transport in this application, or data throughput, is based on the impact of Internet random delay through a link, and the interaction of a large number of control loops. It is related to first passage percolation in a category of regular random graphs, which contains lattices and trees.

Quite surprisingly, the data throughput can be proved to be positive even for infinite group, under the assumption that delays are light tailed. This result has never been met, to the best of my knowledge, in any other reliable group communication scheme, and it can be shown for very general type of communication graph. The case of heavy tail delays is in particular interesting as the same result holds for very restricted communication topology, under a small moment condition. For general infinite tree, this remains an open problem.

Sourav Chatterjee, Concentration of Haar measures, with an application to random matrices.

We shall present a method for obtaining the concentration of Haar measures on compact groups using information about the rates of convergence to stationarity of associated random walks. The technique will applied to obtain the concentration of the spectrum of sums of random hermitian matrices, giving a quantitative version of Voiculescu's celebrated connection between random matrices and free probability.

Sebastian Cioaba, Eigenvalues, Ramanujan Graphs and Expanders

The explicit construction of Ramanujan graphs requires deep results from number theory, representation theory and arithmetic geometry. We will discuss various aspects regarding the distribution of the eigenvalues of regular graphs including a new elementary proof of a theorem of Serre and a simple way of constructing new expanders from old.

Codina Cotar, Some new results on edge reinforced random walk

We consider reinforced random walk, in which the transition probabilities at each step are influenced by the number of crossings of the edges. For finite graphs we outline a new approach to study the behaviour for the proportion of time spent on the individual edges. Previously, an asymptotic formula was derived by S. Rolles and M.S.Keane. We find an explicit formula for this. Some applications of this new approach are also given.

Lee Dicker, Partitioning sites of $\mathbb{Z}^3$ according to which random walk gets there first.

Consider two independent simple random walks $X$ and $Y$ on lattice points of the $d$-torus. A point is painted red if $X$ reaches it first and blue if $Y$ reaches it first. Let $W$be the difference in number of red points and blue points. We find an upper bound (though not the best) on the second moment of $W$. Here we restrict our attention to the case $d = 3$.

Christian Gromoll, Measure valued processes and queues

This talk will introduce several related measure valued stochastic processes; in each case the process models a "resource sharing" type of queueing system that is difficult to analyze using finite dimensional stochastic processes. By considering these examples together, the talk aims to give an overview of the measure valued approach to such queueing models. The discussion will highlight some techniques and intuition for obtaining scaling limits, as well as describe some recent results and open questions.

Majid Hosseini, A Sharp Inequality for the First Exit Time of Brownian Motion

We will prove an inequality for the first exit time of Brownian motion from a bounded planar domain that is symmetric with respect to one of the coordinate axes and convex with respect to the other axis. This inequality is derived from a similar inequality for random walks. The basic tools involved are coupling and invariance principle.

Liza Jones, Number Variance Saturation

The number variance of a point process on the real line is the variance of the number of points observed in a given interval. I will discuss how this statistic relates to the distribution of the Riemann zeros on the critical line, random matrix theory and systems of one-dimensional Brownian motions.

G. Jay Kerns, Exchangeability and de Finetti's Theorem

The celebrated theorem of de Finetti says that an infinite sequence of exchangeable random variables is a mixture of independent and identically distributed random variables. The result happens to fail in general for sufficiently pathological state spaces and for some finite sequences, in particular, for sequences that are negatively correlated. A geometric argument will be used to provide insight into the reason for the failure, and to simultaneously suggest that one needs only to consider an extended notion of "mixture" to recapture de Finetti's elegant representation. Finally, connections to Bayesian consistency and Statistical Physics will be examined.

Panki Kim, Relative Fatou's theorem for $(-\Delta)^{\alpha/2}$-harmonic functions in bounded $\kappa$-fat open sets

In this talk, we will discuss about the boundary behavior of $(-\Delta)^{\alpha/2}$-harmonic functions (equivalently harmonic function for symmetric $\alpha$-stable processes) in bounded $\kappa$-fat open set where $\alpha \in (0,2)$. Consider a positive $(-\Delta)^{\alpha/2}$-harmonic function $u$ in a bounded $\kappa$-fat open set $D$, and a positive $(-\Delta)^{\alpha/2}$-harmonic function in $D$ vanishing outside of $D$. It is true that non-tangential limits of $u/h$ exist almost everywhere with respect to the Martin-representing measure of $h$. We also study relative Fatou's theorem for operators obtained from the generator of the killed $\alpha$-stable process in bounded $\kappa$-fat open set $D$ through non-local Feynman-Kac transforms.

George Kordzakhia, A predator-prey model on homogeneous trees

There are two types of particles interacting on a homogeneous tree of degree $d+1$. The particles of the first type colonize the empty space with exponential rate 1, but cannot take over the vertices that are occupied by the second type. The particles of the second type spread with exponential rate $\lambda$. They colonize the neighboring vertices that are either vacant or occupied by the representatives of the opposite type, and annihilate the particles of the type 1 as they reach them. There exists a critical value $\lambda_c = (2d-1)+\sqrt{(2d-1)^2-1}$ such that the first type survives with positive probability for all $\lambda < \lambda_c$, and dies out with probability one for $\lambda > \lambda_c$. We also find the growth profile which characterizes the rate of growth of the type 1 in the space-time on the event of survival.

Manjunath Krishnapur, Two Dimensional Determinantal Processes

There is a one parameter family of invariant determinantal processes, one each on the Complex plane, the sphere and the unit disk. We introduce these ensembles, and show how (some of them) occur in Random Matrix Theory/Random Analytic Function Theory. We also indicate an application of the spherical ensembles to the problem of distributing points uniformly on a sphere (this part is jointly with Balint Virag)

Amaury Lambert, The logistic branching process, coalescence with fragmentation, and fixation of mutant alleles

Nicolas Lanchier, Epidemics with recovery on a lattice of social cluster

The first epidemic model including a spatial structure in the form of local interactions is the contact process. The aim of this talk is to investigate an extension of the contact process describing the course of a single disease within a spatially structured population distributed in social clusters. The evolution of the disease depends on three parameters, namely the outside infection rate which models the interactions between the clusters, the within infection rate which takes into account the repeated contacts between individuals in the same cluster, and the size of each of the social clusters. We will investigate the existence of nontrivial stationary distributions for the stochastic process depending on the value of each of the three parameters of the system.

Joint work with Lamia Belhadji.

Greg Lawler, The Laplacian random walk

The Laplacian-$b$ random walk is a random walk on the integer lattice whose transitions probabilties are weighted by the $b$th power of the probability that simple random walk avoids the path up to that point. I will explain why it is conjectured that the scaling limit of the Laplacian walk in two dimensions is a Schramm-Loewner evolution (SLE) with parameter $\kappa = \kappa(b)$.

Manuel Lladser, Multiple pattern frequencies for Markovian sequences.

A new method to compute the distribution of the number of occurrences of various patterns in a random string is presented. We first show how to determine the distribution of the number of occurrences of none, some, or all of a list of patterns in a random string generated by a memoryless source. We then show how to handle the more general case in which the random string is produced by a Markov process with a bounded amount of memory. The key idea of our method is based on the observation that the probability of occurrence of a single pattern in a random string of length N corresponds to the probability that a derived Markov chain visits certain terminal state within N steps. Through a synchronization argument we show how the probability of occurrence of various patterns can be formulated in terms of a first-order Markov chain with a state space corresponding to lattice points in a hypercube. The computation of the transition matrix associated to this chain can be implemented automatically with some few routines. Within this framework the use of generating functions and linear algebra methods are readily available to determine explicitly the probability of simultaneous occurrence of various patterns in a random string of arbitrary length. The method has applications in various areas of biology but mainly problems related to genome-wide searches in RNA. In this context, a crucial problem is to assess the statistical significance of the matches: how often would be the same set of interacting modules (patterns) be found just by chance?

Jean Mairesse, Random walks on groups, braids, and Tetris heaps

Consider a transient nearest neighbor random walk on a group or monoid X with finite set of

generators S. The goal is to obtain an explicit description of the "harmonic measure" which gives the direction of escape to infinity of the walk. The drift of the walk, or speed of escape to infinity, as well as the entropy can then be obtained as a by-product. When X is a free group, it is known that the harmonic measure has a simple Markovian structure (Dynkin & Malyutov, 61; Sawyer & Steger, 87). We show that it is still possible to give a simple and explicit combinatorial description of the harmonic measure (almost Markovian) for a much larger class of groups or monoids with a tree-like Cayley graph. This includes zero-automatic pairs, free products with amalgamation, HNN extensions. We also show how to adapt the general results to compute explicitly: (i) the entanglement speed of a random braid on three strands (the group $< a, b | aba = bab >$); (ii) the growth speed of some random Tetris heaps (e.g. the group $< a, b, c | ab = ba >$).

Tai Melcher, Malliavin calculus and heat kernel inequalities

Malliavin calculus is an infinite-dimensional differential calculus on Wiener space. The original motivation for its development was to produce a probabilistic proof of the celebrated Hörmander Theorem, which states that solutions to certain SDEs have smooth transition densities. Malliavin calculus, also known as stochastic calculus of variations, has since transcended this application and inspired many new results in stochastic analysis and other fields of mathematics.

I would like to give a flavor of this analysis on path space by introducing the two of the three fundamental objects of Malliavin calculus: the gradient, or derivative, operator and the divergence, or integral, operator. (The third fundamental object, the Ornstein-Uhlenbeck operator, can be defined as a composite of these actions.) I will also briefly describe an application to heat kernel inequalities that demonstrates the usefulness of this theory.

Mathieu Merle, Estimates for the probability of hitting far points for the voter model

The voter model is one of the standard interacting particle systems. Here, we will try to investigate the asymptotic order of the probability for a voter model to hit a far point. We consider on $\mathbb{Z}^d$ a two-type voter model $\xi_t^0$ initially with a single 1 at point 0. What is the asymptotic order of the probability for $\xi_t^0$ to hit the point c.x, with c going to infinity ? We will see that for small dimensions ($d$=2 or 3), this probability is of the same order of the probability for $\xi_t^0$ to survive until a time of order $c^2$, that is $\ln(c)c^{-2}$ for $d = 2, c^{-2}$ for $d = 3$. This is related to the fact that the scaling limit of $\xi_t^0$ is the excursion measure of super-Brownian motion on $\mathbb{R}^d$. For d¿4, the probability of hitting c.x for $\xi_t^0$ will be of order $c^{2-d}$ (while the probability for $\xi_t^0$ to be alive at time of order $c^2$ remains of order $c^{-2}$). This will be done by a moment method. For $d = 4$, we conjecture the probability of hitting c.x to be of order $c^{-2}\ln(c)^{-1}$.

Carl Mueller, The speed of a random traveling wave

We describe work in progress with Leonid Mytnik and Jeremy Quastel. The KPP equation is a standard model for the study of traveling waves. A large class of initial conditions yield solutions which converge to a limiting shape, which moves with constant velocity. Adding noise to the equation may give a stationary ensemble of shapes, with an average speed which is different than the speed of the deterministic wave. Brunet and Derrida have conjectured some surprising results about the speed of the wave in the random case, when the noise is small. Conlon and Doering have given an inequality which verifies half of the conjecture. We describe an approach which, if successful, will give the other half of the conjecture.

Peter T. Otto, Limit Theorems for a Class of Mean-Field Models with Varying

For a class of mean-field models that includes the classical Curie-Weiss model and the mean-field Blume, Emery, Griffiths model, for fixed temperature, it has been shown that under appropriate

assumptions there exists a positive real number $\lambda$ and a positive integer $k$ such that $S_n/n^{1-2k}$ converges weakly to a random variable with density proportional to $\exp(\lambda x^{2k}/(2k)!)$, where $S_n$ is the partial sum for triangular arrays of dependent random variables. In this talk, I will discuss the relationship of these results to the continuous second-order phase transition exhibited by these models and then extend the results to the case where the temperature converges to the phase transition critical value as $n \to \infty$. We show that there exists a critical speed for the temperature at which the limiting distribution of the scaled partial sums changes from a density proportional to $\exp(\lambda x^2/2)$ to one proportional to $\exp(\lambda x^{2k}/(2k)!)$. At the critical speed, the limiting distribution becomes a combination of these two distributions.

Alexander Roitershtein, LLN and CLT for multi-type branching processes with immigration in a random environment and for random walks in random environment on a strip.

Abstract: We consider a subcritical multi-type branching process with immigration in a random environment $Z(n)$. By using the existence of a limiting distribution for $Z(n)$ (proved by E. Key) we obtain a LLN and a CLT for its partial sums and study the asymptotic behavior of the random segments $Z(v_n+1), ..., Z(v_{n+1})$, where $v_n$ are successive return times to a fixed recurrent point. We also discuss related LLN and CLT for random walks in random environment on a strip.

A. Rudas, Random trees and general branching processes

Cornelia Sauer, Stochastic networks with unreliable nodes

We model and analyse networks of queues in which the servers at the nodes are unreliable. These models have various applications in production and telecommunication systems. We incorporate into the Markovian system description a rather general breakdown and repair behaviour of the nodes, in particular the up and down times of the servers need not be exponential. Nodes may break down as an isolated event or in groups simultaneously. Customer routing connected with nodes in down status is regulated by rules originating from the theory of information blocking.

This modelling approach leads to explicit stationary distributions of product form. From these the standard performance measures of the network can be computed. Moreover we derive conditional job observer properties by introducing a point process description of the customer processes using Palm measures to describe the state distributions just before and just after selected event moments .

Rinaldo Schinazi, Mathematical Models for Virus Dynamics

We introduce a spatial stochastic model for virus dynamics. We show that if the death rate of infected cells increases too fast with the virus load the virus dies out. This is in sharp contrast with what happens in the (non-spatial deterministic) basic model for virus dynamics.

Nathan Shank, Nearest Neighbor Graphs on Cantor Sets

Consider a Poisson number of points uniformly distributed on the Cantor set. We can construct a nearest neighbors graph just as in the Euclidean setting. A major question is what is the asymptotics for the expected total edge length of this nearest neighbors graph? Does it scale similar to the Euclidean case? Is there even a limit as the intensity increases? My recent work shows the appropriate asymptotics and shows that the limit can not exist! The limit has a log periodic fluctuation which does not appear in the Euclidean case.

Florian Sobieczky, Random Walks on the finite components of random Partial Graphs of Transitive Graphs

The expected n-step return-probability of a random walk with symmetric transition probabilities on a random partial graph of a regular graph of degree $d$ with transitive automorphism group is considered. The law of the random edge-set is assumed to be stationary with respect to some transitive subgroup of the automorphism group. By the spectral theory of finite random walks, bounds in terms of the expected number of open clusters per vertex and moments of the cluster size are obtained. The result shows that the return probability has an upper bound converging at the same rate as that of the random walk conditioned on families of finite clusters $C$ with mixing time of the order of $|C|^y$, where $y <= 2 - 2/(d+1)$.

Vishal Sood, Voter Model On Heterogeneous Graphs

We study the voter dynamics model on heterogeneous graphs. We exploit th non-conservation of the magnetization to characterize how consensus is reached. For a network of $N$ nodes with an arbitrary but uncorrelated degree distribution, the mean time to reach consensus $T_N$ scales as $N\mu_1^2/\mu_2$, where $\mu_k$ is the $k^{\text{th}}$ moment of the degree distribution. For a power-law degree distribution $n_k \sim k^{-\nu}$, $T_N$ thus scales as $N$ for $\nu > 3$, as $N/\ln N$ for $\nu = 3$, as $N^{(2\nu-4)/(\nu-1)}$ for $2 < \nu < 3$, as $(\ln N)^2$ for $\nu = 2$, and as $\mathcal{O}(1)$ for $\nu < 2$. These results agree with simulation data for networks with both uncorrelated and correlated node degrees.

Gabor Szabo, Geometry and degeneracy of minimum spanning trees on random networks

The minimum spanning trees on scale-free graphs are shown to be scale-free as well, in the presence of random edge weights. The probability distribution of the weights on the tree differs from regular lattices reflecting the typically short distances (small-world property). We consider also the trees in the absence of such randomness and the ensuing massive degeneracy, which is analyzed with graph theoretical arguments.

David Welch, Building and fitting models of coupled host-virus genealogies

We treat a host population interacting with a virus. The virus is transmitted both horizontally and vertically. We modify the Moran model to describe the stochastic dynamics of individual host and viral lineages. The associated diffusion process, obtained in the limit of large host population, is related to the Neuhauser-Krone selection graph process. Model components representing decreased fertility due to infection and transmission by contact lead to a joint genealogy graph containing both branching and coalescing nodes. We study an isolated population of cats infected with FIV. We fit the joint host-virus process to microsatellite host sequence and viral base sequence data gathered from the cat and FIV populations. We use MCMC to average over the variable dimension parameter space of labeled graphs.

Feng Zhang, Harmonic functions of subordinate killed Brownian motion in exterior domain

We study harmonic functions of subordinate killed Brownian motion in exterior domain, completement of bounded $C^{1,1}$ domain. The structure of harmonic functions will be given and both Martin boundary and minimal Martin boundary are the Euclidean boundary $\partial D$ and infinity. The decomposition of harmonic functions are given, which tells any harmonic function is sum of purely excessive function and a function which is invariant under the semigroup. We also show Harnack principle for positive harmonic functions of subordinate killed Brownian motion.

Xinyi Zhang, Expected length of minimum spanning tree

Inspired by the random assignment problem, we are pursuing the exact formula for the expectation of length of the minimum spanning tree (MST) of a connected graph with independent and

identical edge distribution. An exact formula in terms of Tutte polynomial is given and this generalizes Steele's formula in the uniform case. As a consequence, we show that for any connected graph, the expected length of MST is shorter in the uniform $(0,1)$ case than in the exponential rate one case. Explicit and asymptotic evaluations are carried out for wheel graph. Deeper properties such as variance and CLT are still under research.

Ou Zhao, Laws of the iterated logarithm for functionals of Markov Chains

Abstract: Let $X_0, X_1, .....X_n, ...$ be an ergodic stationary markov chain in a polish space with stationary distribution $\pi$, let $g$ be a functional of second moments with respect to $\pi$. LIL (Hartman-Wintner's type) will be derived for the partial sum $S_n = g(X_1) + ... + g(X_n)$ under the growth condition $||E[S_n|X_0]|| = O(n^\alpha)$ when $\alpha < 1/2$. The conections with classical LILs will also be discussed. The proof is based on martingale approximation, then the treatment of the remainder term involves ergodic theory and operator theory.

Xiaowen Zhou, Exit problems for reflected spectrally negative Lévy processes

For a spectrally negative Lévy process $X$ on the real line, let $S$ be its supremum process and let $I$ be its infimum process. For $a > 0$ let $\tau(a)$ and $\kappa(a)$ be the times when the reflected processes $Z := S - X$ and $Y := X - I$ first exit level $a$ respectively. Our main results concern the distributions of $(\tau(a), S_{\tau(a)}, Z_{\tau(a)})$ and $(\kappa(a), Y_{\kappa(a)})$. They generalize those known results on Brownian motion. But our approach is different. It relies heavily on Bertoin's results concerning the solution to the so-called two-sided exit problem for $X$. Such an approach can also be applied to study the excursions for the reflected processes.

Fabio Zucca, The discrete integral maximum principle and estimates of transition probabilities

If space and time are discrete the Heat Kernel is deeply connected with the (infinite) matrix of transition probabilities of a reversible random walk. We introduce the discrete analogous of the well-known integral maximum principle and we show how it allows us to estimate the transition probabilities under mild assumptions.