Solutions to Assignment 5

Barr 3.1, Pages 185-186, 2, 3, 10

2. (a) 1 000 000, (b) 10 000 001, (c) 11 101, (d) 100 011 111.

3.

(a) Notice that $2^{16} = 65536 < 72891$ and $2^{17} = 131072 > 72891$, thus 17 digits will be needed.

(b) Notice that $2^{23} = 8,388,608 < 10,643,522$ and $2^{24} = 16,777,216 > 10,643,522$, thus 24 digits will be needed.

10.

(a) a + b = 110,000 , a – b = 110

(b) a + b = 101,111 , a – b = 11,001

(c) a + b = 1,110,100 , a – b = 10

(d) a + b = 10,000,100 , a – b = 1,111,010

E1. Do the following binary arithmetic.

$1100_2 + 1001_2 = 10101_2$

$1100_2 - 1001_2 = 11_2$

$1100_2 \times 1001_2 = 1101100_2$

E2. Rewrite these arithmetic problems and solutions base 10.

$1100_2 = 12_{10}$, $1001_2 = 9_{10}$, So

$12_{10} + 9_{10} = 21_{10}$

$12_{10} - 9_{10} = 3_{10}$

$12_{10} \times 9_{10} = 108_{10}$

E3. Rewrite these arithmetic problems and solutions base 16.

$12_{10} = C_{16}$, $9_{10} = 9_{16}$

$C + 9 = 21_{10} = 1 \times 16^1 + 5 \times 16^0 = 15_{16}$

$C - 9 = 3_{10} = 3 \times 16^0 = 3_{16}$

$C \times 9 = 108_{10} = 6 \times 16^1 + 12 \times 16^0 = 6C_{16}$.

E4. Ditto E1-3 for $1100_2 / 1001_2$.

i)

$$
\begin{array}{r}
1.0101\ldots \\
1001 \overline{)\,1100} \\
\underline{1001} \\
1100 \\
\underline{1001} \\
1100 \\
\underline{1001} \\
11
\end{array}
$$

and so on.

Thus, $1100_2 / 1001_2 = 1.0101\ldots = 1.\overline{01}$.

ii) $1100_2 = 12_{10}$, $1001_2 = 9_{10}$.

$$
\begin{array}{r}
1.33\ldots \\
9 \overline{)\,12} \\
\underline{9} \\
30 \\
\underline{27} \\
30 \\
\underline{27} \\
3
\end{array}
$$

and so on

Thus, $12_{10} / 9_{10} = 1.33\ldots = 1.\overline{3}$.

iii) $12_{10} = C_{16}$, $9_{10} = 9_{16}$.

First, note the multiplication of 9 in base 16:

$$
\left.
\begin{array}{l}
9 \times 1 = 9 \times 16^0 = 9_{16} \\
9 \times 2 = 1 \times 16 + 2 = 12_{16} \\
9 \times 3 = 1 \times 16 + 11 = 1B_{16} \\
9 \times 4 = 2 \times 16 + 4 = 24_{16} \\
9 \times 5 = 2 \times 16 + 13 = 2D_{16} \\
9 \times 6 = 3 \times 16 + 6 = 36_{16}
\end{array}
\right\} \text{Using This:}
\qquad
\begin{array}{r}
1.55\ldots \\
9 \overline{)\,C} \\
\underline{9} \\
30 \\
\underline{2D} \\
30 \\
\underline{2D} \\
3
\end{array}
$$

3 and so on

Remember the '30' here $= 30_{16} = 48_{10}$, so $9 \times 5 = 45_{10} = 2D_{16}$ is the closest multiple of 9 less than 30_{16}.

Thus, $C/9 = 1.55\ldots = 1.\overline{5}$.

E5. Conversion from base 2 and base 16 is especially straightforward since 16 is a power of 2 i.e. $16 = 2^4$.

Thus, to convert a number a number from base 2 to base 16, starting from the right, group 4 bits into 1 and convert them into a single hexadecimal.

An Example:

Convert 110101111_2 into base 16.

Grouping we obtain 1, 1010 and 1111.
$1_2 = 1_{10} = 1_{16}$
$1010_2 = 10_{10} = A_{16}$
$1111_2 = 15_{10} = F_{16}$

So 110101111_2 = 1AF in hexadecimal.

Suppose we wanted to convert 11010111_2 into base 10:

$110101111_2 = 1 \times 2^8 + 1 \times 2^7 + 0 \times 2^6 + \ldots + 1 \times 2^0 = 431_{10}$.

Note that as the number of digits increase, converting from base 2 involves calculating larger and larger powers of 2. Given a 100-bit number, for example, it is much simpler to divide the number into bits of 4 and convert them into a single hexadecimal, than it is to calculate 2^{100}, 2^{99}, etc.

E6. 1 kilobyte equals $2^{10} = 1024$ bytes.
$1024 = 4 \times 16^2 + 0 \times 16^1 + 0 \times 16^0$, so $1024 = 400_{16}$.