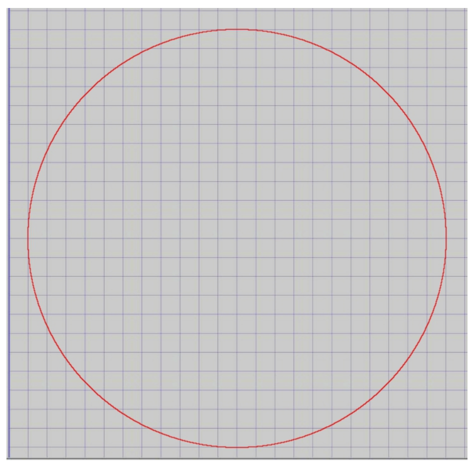
Consider the 2-dimensional random walk defined by $(X_0, Y_0) = (0, 0)$ and $(X_{n+1}, Y_{n+1}) = (X_n, Y_n) + \xi_n$ where $\xi_1, \xi_2, \ldots$ are independent with

$$P(\xi_n = (1, 0)) = P(\xi_n = (-1, 0)) = P(\xi_n = (0, 1)) = P(\xi_n = (0, -1)) = \frac{1}{4}.$$

1. Find a number a such that $M_n = X_n^2 + Y_n^2 - an$ is a martingale.

We'd like to estimate the expected time ET where T is the time it takes to exit a disk of radius r. In other words, $T = \min\{n \geq 0 : X_n^2 + Y_n^2 > r^2\}$.



2 Estimate $E(X_T^2 + Y_T^2)$ when $r = 10$. (Give upper and lower bounds.)

3 Use parts 1 and 2 to estimate ET.

4 How does your estimate generalize to (a) general radius r? (b) higher dimensions?