Flip a fair coin until it comes up heads twice in a row or tails twice in a row.

We can model this process as a Markov chain with states $\emptyset, H, T, HH, TT$. For example, state H means that the most recent flip was heads but we have not yet achieved two heads in a row or two tails in a row. State HH is absorbing and means we have achieved two heads in a row. The chain starts in state $\emptyset$ and transitions from there to either H or T depending whether the first flip is heads or tails.

1. Write down the transition matrix for this chain.
2. List the closed sets. Which ones are irreducible?
3. Which states are recurrent?
4. Find $r_{H,T}$ and $r_{T,T}$.
5. Given that the first flip is heads, what is the expected number of times the chain visits state T?